Unit 5
Recursive Functions

AFM
Mrs. Valentine

5.1 Sequences

- Objective: I will be able to identify an infinite series. I will be able to read and write recursion formulas. I will be able to graph sequences.

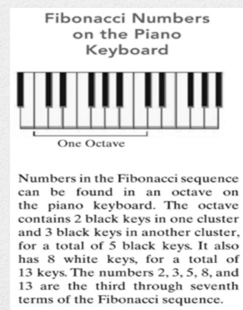
- Vocabulary

Fibonacci sequence	General Term	Recursion Formulas	Finite Sequence	Graph of a sequence
Infinite Sequence				

5.1 Sequences

- Sequences
  - Many creations in nature involve intricate mathematical designs, including a variety of spirals.
  - The Fibonacci sequence can be found in nature in many places.
    - The sequence begins with two ones and every term thereafter is the sum of the two preceding terms
    - 1, 1, 2, 3, 5, 8, 13, 21, 24, 55, 89, 144, 233, … a_n
  - a_n is a general term of the sequence, also called the nth term.

5.1 Sequences



5.1 Sequences

- An infinite sequence is a function whose domain is the set of positive integers. The function values, or terms, of the sequence are represented by

$$a_1, a_2, a_3, a_4, \ldots , a_n, \ldots$$

Sequences whose domains consist only of the first n positive integers are called finite sequences.

- Because a sequence is a function whose domain is the set of positive integers, the graph of a sequence is a set of discrete points (not connected).

5.1 Sequences

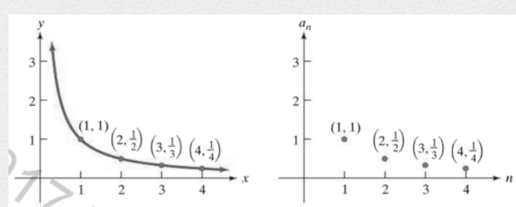


FIGURE 11.1(a) The graph of $f(x) = \frac{1}{x}, x > 0$

FIGURE 11.1(b) The graph of $\{a_n\} = \left\{\frac{1}{n}\right\}$

5.1 Sequences

- Recursion Formulas
  - A recursion formula defines the nth term of a sequence as a function of the previous term.
  - Example: $a_n = 3a_{n-1} + 2$
  - Recursion formulas can be used to find the next term in a sequence, but you must already know the previous term in order to find it.

5.2 Summary Notation

- Objective: I will be able to read, write, and evaluate factorial notation. I will be able to read, write, and manipulate summation notation for the sums of a sequence.
- Vocabulary

Factorial Notation	Summation Notation	Index of Summation	Upper Limit of Summation	Lower Limit of Summation
Expanding the Summation Notation				

5.2 Summary Notation

- Factorial Notation
  - Products of consecutive positive integers occur quite often in sequences.
  - Factorial notation is a special notation for consecutive products.
    - If n is a positive integer, the notation n! (Read "n factorial) is the product of all positive integers from n down through 1.

$$n! = n(n-1)(n-2)\ldots(3)(2)(1)$$

    - 0! = 1 by definition

5.2 Summary Notation

- Like exponents, factorials only affect the number they are directly following unless grouping symbols, like parentheses, appear.
  - $2 * 3! = 2(3 * 2 * 1) = 12$
  - $(2 * 3)! = 6! = 6 * 5 * 4 * 3 * 2 * 1 = 720$

5.2 Summary Notation

- It can be useful to find the sum of the first n terms of a sequence.
- If there are too many terms, writing out the entire sum can be cumbersome.
- Summation notation can be used to simplify writing sums of a sequence.
  - The sum of the first n terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$$

5.2 Summary Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$$

- *i* is the index of summation, which indicates the domain of the terms of the sequence being added.
- *n* is the upper limit of summation (the largest value of the index)
- *1* is the lower limit of the summation (the smallest value of the index)

5.2 Summary Notation

- NOTES:
  - Any letter can be used for the index
  - The lower limit can be other numbers than 1
- Writing out the sum of the terms is called expanding the summation notation

5.2 Summary Notation

- Properties of Sums

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

5.3 Arithmetic Sequences

- Objective: I will be able to recognize an arithmetic sequence and graph its function. I will be able to find the common difference fro an arithmetic sequence and give a set of values for the sequence.
- Vocabulary

Arithmetic Sequence	Common Difference	General Term of Arithmetic Sequence	

5.3 Arithmetic Sequences

- Arithmetic Sequences
  - An arithmetic sequence is a sequence in which each term after the first differs from the preceding term by a constant amount.
  - The difference between consecutive terms is called the common difference of the sequence.

5.3 Arithmetic Sequences

- Finding the Common Difference for an Arithmetic Sequence
  - Represented by the symbol *d*.
  - Subtract any two consecutive points in an arithmetic sequence to find the common difference.

$$d = a_n - a_{n-1}$$

5.3 Arithmetic Sequences

- An arithmetic sequence is a linear function whose domain is the set of positive integers
  - The graph of this function is a set of discrete points (points not connected)
  - The points lie on a straight line.
- Writing the Terms of an Arithmetic Sequence
  - To write a set of terms in an arithmetic sequence, you must know at least one term, usually a_1, and the common difference.
  - Use the recursion formula $a_n = a_{n-1} + d$

5.4 Arithmetic Sums

- Objective: I will be able to find a specific term within an arithmetic sequence. I will be able to find the sum of the first n terms of an arithmetic sequence.
- Vocabulary

Sum of the First n Terms of Arithmetic Sequence	

5.4 Arithmetic Sums

- The General term of an Arithmetic Sequence
  - Consider how the first six terms in an arithmetic sequence are determined:

a_1,	$a_1 + d$,	$a_1 + 2d$,	$a_1 + 3d$,	$a_1 + 4d$,	$a_1 + 5d$.
a_1, first term	a_2, second term	a_3, third term	a_4, fourth term	a_5, fifth term	a_6, sixth term

  - The nth term (general term) of an arithmetic sequence with the first term a_1 and common difference d is

$$a_n = a_1 + (n-1)d$$

5.4 Arithmetic Sums

- The Sum of the First n Terms of an Arithmetic Sequence
  - The sum of the first n terms of an arithmetic sequence, denoted by S_n and called the nth partial sum, can be found without having to add up all the terms.
  - The sum, S_n of the first n terms of an arithmetic sequence is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

5.5 Geometric Sequences

- Objective: I will be able to recognize geometric sequences. I will be able to find the common ratio and a set of terms for a geometric sequence. I will be able to find the general term of a geometric sequence.
- Vocabulary

Geometric Sequence	Common Ratio	General Term of a Geometric Sequence

5.5 Geometric Sequences

- Geometric Sequences
  - A geometric sequence is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed nonzero constant.
  - The amount by which we multiply each term is called the common ratio of the sequence.

5.5 Geometric Sequences

- Find the Common Ratio of a Geometric Sequence
  - The common ratio r is found by dividing any term after the first term by the term that directly precedes it.

$$r = \frac{a_n}{a_{n-1}}$$

  - A geometric sequence with a positive common ratio other than 1 is an exponential function whose domain is the positive set of integers.

5.5 Geometric Sequences

- Write Terms of a Geometric Sequence
  - To write a set of terms in a geometric sequence, you must know at least one term, usually a_1, as well as the common ratio.
  - Use the recursion formula: $a_n = a_{n-1} * r$
- The General Term of a Geometric Sequence
  - The nth term (the general term) of a geometric sequence with the first term a_1 and the common ratio r is

$$a_n = a_1 r^{n-1}$$

5.6 Geometric Series

- Objective: I will be able to find the sum of n terms of a geometric sequence. I will be able to understand and find values of an annuity. I will be able to use geometric series and their sums to find multiplier effect values on the economy.
- Vocabulary

*n*th Partial Sum	Annuity	Value of the Annuity	Multiplier Effect
Infinite Geometric Series		Sum of Infinite Geometric Series	

5.6 Geometric Series

- The Sum of the First n Terms of a Geometric Sequence
  - The sum of the first n terms of a geometric sequence, denoted by S_n and called the nth partial sum, can be found without having to add up all the terms.
  - The sum of the first *n* terms of a geometric sequence is given by

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

  - The common ratio cannot equal 1.

5.6 Geometric Series

- Annuities
  - An annuity is a sequence of equal payments made at equal time periods.
  - An IRA is an example of an annuity.
  - The terms in this sequence are based on the simple interest formula: *A=P1+rt*
  - The value of the annuity is the sum of all deposits made plus all interest paid.

5.6 Geometric Series

- If P is the deposit made at the end of each compounding period for an annuity at r percent annual interest compounded n times per year, the value, A, of the annuity after t years is

$$A = \frac{P\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}$$

5.6 Geometric Series

- Geometric Series
  - An infinite sum in the form

$$a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1} + \cdots$$

with the first term a_1 and common ratio r is called an infinite geometric series.

5.6 Geometric Series

- The sum of an Infinite Geometric Series
  - If $-1 < r < 1$, then the sum of the infinite geometric series above is given by

$$S = \frac{a_1}{1 - r}$$

  - If $|r| \geq 1$, the infinite series does not have a sum.

5.6 Geometric Series

- Multiplier Effect
  - A tax rebate that returns a certain amount of money to taxpayers can have a total effect on the economy that is many times this amount.
  - In economics, this phenomenon is called the multiplier effect.
  - This can be found by finding the sum of an infinite geometric series.