Unit 6
Probability

AFM

Valentine

6.1 Binomial Theorem

- Objective: I will be able to read and evaluate binomial coefficients. I will be able to expand binomials using binomial theorem.

- Vocabulary

Binomial Coefficient	Binomial Theorem			

6.1 Binomial Theorem

- Binomial Coefficients
  - For nonnegative integers n and r, with $n \geq r$, the expression $\binom{n}{r}$ (read "n above r") is called the binomial coefficient and is defined by

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

- The symbol ${}_nC_r$ is often used in place of $\binom{n}{r}$ to denote binomial coefficients.

6.1 Binomial Theorem

- Binomial Theorem
  - When we write out $(a + b)^n$, where n is a positive integer, a number of patterns begin to appear.

$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

6.1 Binomial Theorem

- Expanded form of the binomial expression is a polynomial. Observe the following patterns:
  - The first term in the expansion of $(a + b)^n$ is a^n. The exponents decrease by 1 in each successive term.
  - The exponents on b in the expression $(a + b)^n$ increase by 1 in each successive term. In the first term, the exponent on b is 0. The last term is b^n.

6.1 Binomial Theorem

- The sum of the exponents on the variables in any term in the expansion of $(a + b)^n$ is equal to n.
- The number of terms in the polynomial expansion is one greater than the power of the binomial, n. There are $n + 1$ terms in the expanded form of $(a + b)^n$.

6.1 Binomial Theorem

- If we use binomial coefficients and the pattern for the variable part of each term, a formula called the binomial theorem can be used to expand any positive integral power of a binomial.

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \cdots + \binom{n}{n} b^n$$

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{(n-r)} b^r$$

6.2 Binomial Expansion

- Objective: I will be able to find a particular term in a binomial expansion. I will be able to use Pascal's Triangle to find the coefficient of a term in a binomial expansion.

- Vocabulary

Pascal's Triangle				

6.2 Binomial Expansion

- Finding a Particular Term in a Binomial Expansion
  - The $(r+1)$st term of the expansion of $(a+b)^n$ is $\binom{n}{r} a^{n-r} b^r$

6.2 Binomial Expansion

- Pascal's triangle
  - Pascal's triangle is an array of numbers showing coefficients of the terms in the expansions of $(a+b)^n$.

Pascal's Triangle

Coefficients in the Expansions

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

6.3 Permutations

- Objective: I will be able to draw and/or read a tree diagram that describes possible combinations of items. I will be able to use the fundamental counting principle to find the number of choices available. I will be able to find the number of permutations for a set.

- Vocabulary

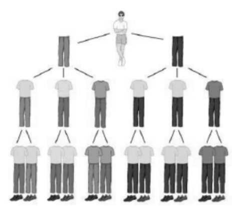
Tree Diagram	Fundamental Counting Principle	Permutations

6.3 Permutations

- Fundamental Counting Principle
  - A tree diagram is a diagram with branches showing the possible combinations of items.
  - The fundamental counting principle states that the number of ways in which a series of successive things can occur is found by multiplying the number of ways in which each thing can occur.

6.3 Permutations

- Example: A woman is trying to decide what to wear. She can choose between blue or black pants, a white, yellow, or blue shirt, and black or red shoes. How many different choices of outfit does this woman have?
  - Tree Diagram:



  - Fundamental Counting Principle

2 pants, 3 shirts, 2 pairs of shoes: *2*3*2=12* outfits possible

6.3 Permutations

- Permutations
  - A permutation is an ordered arrangement of items that occurs when:
    - No item is used more than once.
    - The order of arrangement makes a difference
  - Permutations of n Things Taken r at a Time
    - The number of possible permutations if r items are taken from n items is

$$_nP_r = \frac{n!}{(n-r)!}$$

6.4 Combinations

- Objective: I will be able to distinguish between permutations and combinations. I will be able to calculate the number of combinations that are possible for select items from a set.

- Vocabulary

Combination			

6.4 Combinations

- Combinations
  - A combination of items occurs when
    - The items are selected from the same group.
    - No item is used more than once.
    - The order of the items makes no difference.
  - Difference between permutation and combination:
    - Permutation – order matters
    - Combination – order makes no difference

6.4 Combinations

- Formula for Combinations
  - $_nC_r$ means the number of combinations of n things taken r at a time.
  - The number of possible combinations if r items are taken from n items is

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

  - This is the same formula for the binomial coefficient $\binom{n}{r}$

6.5 Probability

- Objective: I will be able to calculate empirical and theoretical probabilities. I will be able to determine the probability of an event not occurring.

- Vocabulary

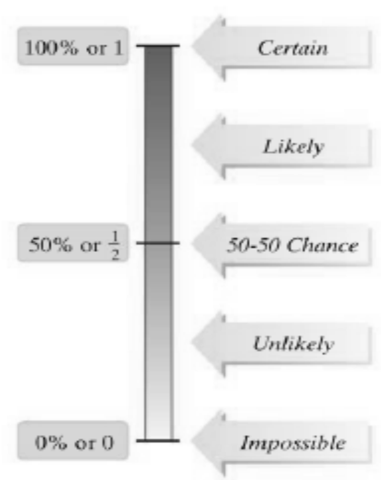
Empirical Probability	Experiment	Sample Space	Theoretical Probability	

6.5 Probability

- Empirical Probability
  - Probabilities of events are expressed as numbers ranging from 0 to 1 (or 0% to 100%).
    - Closer to 1 – event more likely to occur
    - Closer to 0 – event less likely to occur
  - Empirical probability applies to situation in which we observe how frequently an event occurs.
    - The empirical probability of event E, denoted by P(E) is

$$P(E) = \frac{observed\ number\ of\ times\ E\ occurs}{total\ number\ of\ observed\ occurences}$$

6.5 Probability



Possible Values for Probabilities

6.5 Probability

- Theoretical Probability
  - Any occurrence for which the outcome is uncertain is called an experiment.
  - The set of all possible outcomes of an experiment is the sample space of the experiment, denoted by S.
  - An event, denoted by E, is any subcollection, or subset, of a sample space.

6.5 Probability

- Theoretical probabilities applies to situations in which the sample space only contains equally likely outcomes, all of which are known.
  - If an event E has n€ equally likely outcomes and its sample space S has n(S) equally likely outcomes, the theoretical probability of event E, denoted by P(E), is

$$P(E) = \frac{number\ of\ outcomes\ in\ event\ E}{number\ of\ outcomes\ in\ sample\ space\ S} = \frac{n(E)}{n(S)}$$

6.5 Probability

- Probability of an Event Not Occurring
  - If we know P(E), the probability of an event E, we can determine the probability that the event will not occur, denoted by P(not E).
  - Because the sum of the probabilities of all possible outcomes in any situation is 1, the probability that an event E will not occur is equal to 1 minus the probability that it will occur.

$$P(not\ E) = 1 - P(E)$$

6.6 Probability of Multiple Events

- Objective: I will be able to determine the probability of two events occurring if they are mutually exclusive events, not mutually exclusive events, and/or independent events.

- Vocabulary

Mutually Exclusive Events	Independent Events	

6.6 Probability of Multiple Events

- *Or* Probabilities with Mutually Exclusive Events
  - If it is impossible for any two events, A and B, to occur simultaneously, they are said to be mutually exclusive.
  - If two events are mutually exclusive, the probability that either A or B will occur is determined by adding their individual probabilities.
    - *P(A or B) = P(A) + P(B)*
    - Set Notation: $P(A \cup B) = P(A) + P(B)$

6.6 Probability of Multiple Events

- *Or* Probabilities That are Not Mutually Exclusive
  - If A and B are events that are not mutually exclusive, the probability that A or B will occur is determined by adding their individual probabilities and then subtracting the probability that A and B will occur simultaneously.
    - *P(A or B) = P(A) + P(B) - P(A and B)*
    - Set Notation: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

6.6 Probability of Multiple Events

- *And* Probabilities with Independent Events
  - Two events are independent events if the occurrence of either of them has no effect on the probability of the other.
  - If two events are independent, we can calculate the probability of the first occurring and the second occurring by multiplying their probabilities.
    - *P(A and B) = P(A) * P(B)*
    - Set Notation: $P(A \cap B) = P(A) * P(B)$

6.7 Expected Values

- Objective: I will be able to make random selections and simulate a model. I will be able to determine the expected value for an outcome and use that information to make the best possible decisions. Students will be able to determine fairness.

- Vocabulary

Random Event	Expected Value	Fair

6.7 Expected Values

- Fairness
  - Fairness is often a matter of opinion.
  - A basic game of chance is considered fair if every player has an equal probability of winning.
  - A choice is fair if all possible options have an equal probability of being chosen.

6.7 Expected Values

- Ex: Two teams decide to play baseball. They want to decide who bats first. Robert and David are the team captains. They each suggest a method to decide who bats first.
  - Robert: Flip a coin. If it lands on heads, my team will bat first. If it lands on tails, David's team bats first.
    - Fair method because there is an equal chance that the coin will land on heads or tails.

6.7 Expected Values

- David: Roll a single die. If it lands on 1, 2, or 3, my team bats first. If the roll is 4, 5, or 6, then Robert's team bats first.
  - Fair method because there is an equal chance of rolling a 1, 2, or 3 as there is to roll a 4, 5, or 6.

- To help eliminate bias, making random selections is a fair way to choose items/people from a set.

6.7 Expected Values

- Making Random Selections
  - You can use probability to make choices and to help make decisions based on prior experience.
  - A random event has no predetermined pattern or bias toward one out come or another.
  - You can use random number tables to help you make fair decisions.

6.7 Expected Values

- Example
  - There are 28 students in a homeroom. Four students are chosen at random to represent the homeroom on a student committee. How can a random number table be used to fairly choose the students?
    - Select a line from a random number table
    - Group the line from the table into two digit numbers.
    - Match the first four numbers less than 28 with the position of the students' names on a list. Duplicates and numbers greater than 28 are discarded because they don't correspond to any student on the list.

6.7 Expected Values

- Making a Simulation
  - Ex: A cereal company is having a promotion in which 1 of 6 different prizes is given away with each box. The prizes are equally and randomly distributed in the boxes of cereal. On average, how many boxes of cereal will a customer need to buy in order to get all 6 prizes.

6.7 Expected Values

- Let the digits from 1 to 6 represent the six prizes.
- Using a graphing calculator, enter the function randInt(1,6) to generate integers from 1 to 6 to simulate getting each prize. One trial is completed when all 6 digits have appeared.
- Count how many boxes of cereal will be bought before all the digits 1 through 6 have appeared.
- Conduct additional trials (19 more for 20 total). Average the results.

6.7 Expected Values

- Calculating an Expected Value
  - Expected value uses theoretical probability to tell you what you can expect in the long run.
  - If you know what *should* happen mathematically, you will make better decisions in problem situations.

6.7 Expected Values

- The expected value is the sum of each outcome's value multiplied by its probability.

If A is an event that includes outcomes $A_1, A_2, A_3, \ldots$ and Value (A_n) is a quantitative value associated with each outcome, the expected value of A is given by

$$\text{Value}(A) = P(A_1) \cdot \text{Value}(A_1) + P(A_2) \cdot \text{Value}(A_2) + \ldots$$

- This is a weighted average.
- Using the expected value is a matter of selecting the choice with the greater expected value.

6.8 Discrete Random Variables

- Objective: I will be able to use discrete random variables to solve probability problems.

- Vocabulary

Discrete Random Variable	Support		

6.8 Discrete Random Variables

- Random Variables
  - Quantities that take on different values depending on chance or probability.
  - Variables whose values are…
    - Number
    - Due to chance
  - Examples
    - # people at a concert
    - # wins of baseball team in a season
    - Height of a student

6.8 Discrete Random Variables

- Discrete Random Variables
  - A set A is countable if either
    - A is a finite set such as {*1,2,3,4*}
    - It can be put in one-to-one correspondence with natural numbers (in this case, the set is said to be countably infinite)
  - A random variable is discrete if its range is a countable set.

6.8 Discrete Random Variables

- Given a random experiment with sample space S, a random variable X is a set function that assigns one and only one real number to each element s that belongs in the sample space S.
- The set of all possible values of the random variable X, denoted x, is called the support, or space, of X.
- NOTE: Capital letters at the end of the alphabet typically represent the definition of the random variable. The corresponding lowercase letters represent the random variable's possible values.