Mrs. Valentine
AFM

UNIT 2
TRIG I

2.1 Angle Measures

- Objective:
  - I will be able to identify angle types, convert between degrees and radians for angle measures, identify coterminal angles, find the length of an intercepted arc, and find linear and angular
- Vocabulary:

Angle	Initial side	Terminal side	Vertex	Standard position
Positive angles	Negative angles	Quadrantal angle	Degrees	Acute angle
Right angle	Obtuse angle	Straight angle	Radian	Central Angle
Radian Measure	Coterminal angles	Linear Speed	Angular Speed	

2.1 Angle Measures

- Recognize and use the vocabulary of angles
  - An angle (α,β,θ, etc.) is formed by two rays that have a common endpoint. One ray is called the initial side and the other the terminal side.

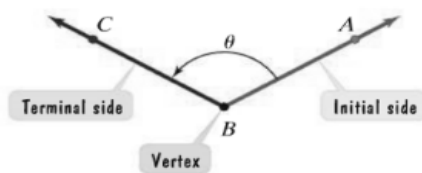


FIGURE 5.2 An angle; two rays with a common endpoint

2.1 Angle Measures

- Standard position: vertex is at the origin of the coordinate system and the initial ray is on the x-axis.
  - Sign
    - Positive angles are generated by rotating counterclockwise.
    - Negative angles are generated by rotating clockwise.

2.1 Angle Measures

- Types
  - If the terminal ray is between axes, it lies in that quadrant.
  - If the terminal ray is ON the x-axis or y-axis, it is a quadrantal angle

2.1 Angle Measures

- Use degree measure
  - If the terminal ray has gone in a complete circle and is back on the initial angle, it has gone 360°.
  - *1°=1/360* complete rotation

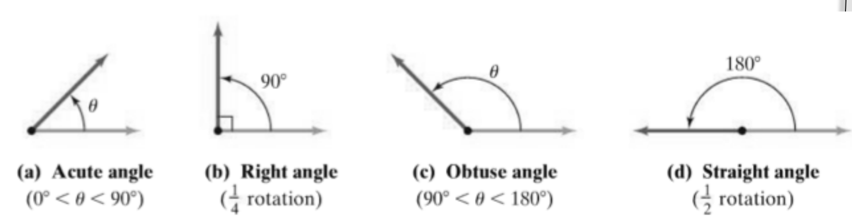


FIGURE 5.5 Classifying angles by their degree measurement

2.1 Angle Measures

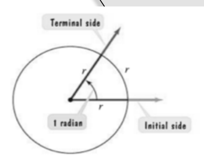
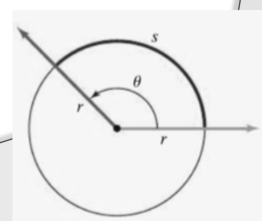


FIGURE 5.6 For a 1-radian angle, the intercepted arc and the radius are equal.

- Using radian measure
  - One radian is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.
  - The radian measure of any central angle is the length of the intercepted arc divided by the circle's radius.

$$\theta = \frac{length\ of\ the\ intercepted\ arc}{radius} = \frac{s}{r} \text{ radians}$$

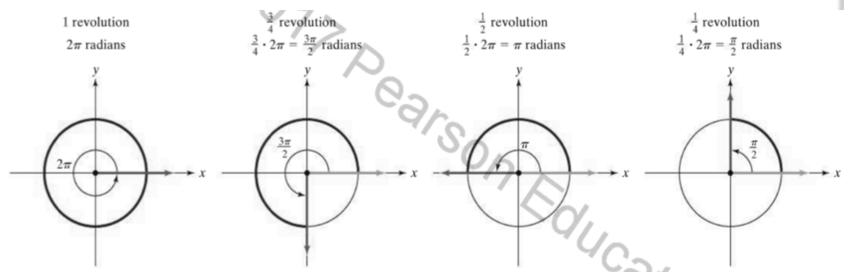


2.1 Angle Measures

- Convert between degrees and radians
  - To convert degrees to radians, multiply degrees by *π radians/180°*
  - To convert radians to degrees, multiply radians by *180°/π radians*

2.1 Angle Measures

- Draw angles in standard position
  - It is important to learn to recognize angles in the coordinate plane by radians – start learning the unit circle!



2.1 Angle Measures

- Find coterminal angles
  - Two angles with the same initial and terminal sides but possibly different rotations are called coterminal angles.
  - Every angle has infinitely many coterminal angles.
    - Increasing or decreasing the degree measure by an integer multiple of 360° (or a radian measure by a multiple of 2π) results in a coterminal angle
    - Positive and negative angles whose absolute values add up to 2π are coterminal angles.

2.1 Angle Measures

- Find the length of a circular arc
  - Can use the radian measure formula $\theta = s/r$ to find the length of the intercepted arc, s.

$$s = r\theta$$

2.1 Angle Measures

- Use linear and angular speed to describe circular motion
  - If a point is in motion on a circle of radius r through an angle of θ radians in time t, then its linear speed is $v = s/t$
  - Its angular speed is $\omega = \theta/t$
  - Linear speed is the product of the radius and the angular speed.

$$s/t = (r\theta)/t = r * \theta/t$$

$$v = r\omega$$

2.2 Right Triangle Trigonometry

- Objective:
  - I will be able to calculate trigonometric functions of acute angles. I will be able to solve for sides and angles of right triangles.

- Vocabulary:

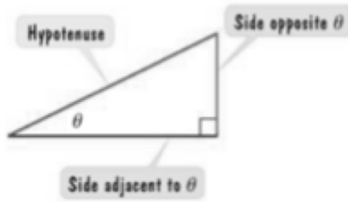
Trigonometry	Hypotenuse	Sine of θ	Cosine of θ	Tangent of θ
Cosecant of θ	Secant of θ	Cotangent of θ	SOHCAHTOA	Pythagorean Theorem

2.2 Right Triangle Trigonometry

- Use right triangles to evaluate trigonometric functions
  - Trigonometry means "measurement of triangles" and is used in navigation, building, and engineering.
  - There are six main trigonometric functions and three inverse functions.
    - Inputs are measures of acute angles in right triangles

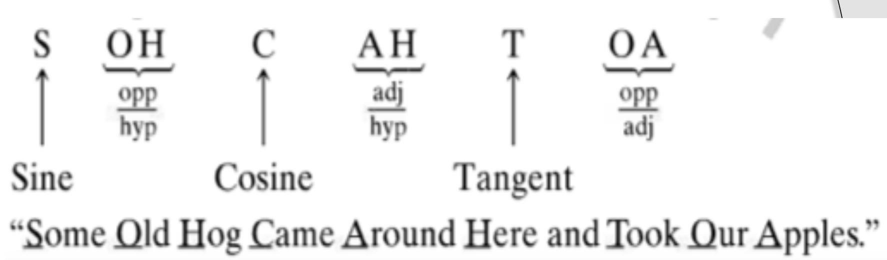
2.2 Right Triangle Trigonometry

- Position of angle in triangle is very important.



- Names of the functions are words rather than letters.
- Functions represent ratios of sides in a right triangle.

2.2 Right Triangle Trigonometry



2.2 Right Triangle Trigonometry

Primary Name	Abbrev.	Reciprocal Name	Abbrev.	Inverse Name	Abbrev.
sine	sin	cosecant	csc	arcsine	$\sin^{-1}$
cosine	cos	secant	sec	arccosine	$\cos^{-1}$
tangent	tan	cotangent	cot	arctangent	$\tan^{-1}$

$$\sin\theta = \frac{Opposite}{Hypotenuse} \qquad \cos\theta = \frac{Adjacent}{Hypotenuse} \qquad \tan\theta = \frac{Opposite}{Adjacent}$$

$$\csc\theta = \frac{Hypotenus}{Opposite} \qquad \sec\theta = \frac{Hypotenus}{Adjacent} \qquad \cot\theta = \frac{Adjacent}{Opposite}$$

$$\sin^{-1}\left(\frac{Opposite}{Hypotenuse}\right) = \theta \qquad \cos^{-1}\left(\frac{Adjacent}{Hypotenuse}\right) = \theta \qquad \sin^{-1}\left(\frac{Opposite}{Adjacent}\right) = \theta$$

2.2 Right Triangle Trigonometry

- Primary and Reciprocal functions are used to find sides while inverse functions are used to find angle measures.
- The trigonometric function values of θ depend only on the size of angle θ and not on the size of the triangle.

2.2 Right Triangle Trigonometry

- Pythagorean Theorem
  - The sum of the squares of the legs of a right triangle equals the square of the length of the hypotenuse ( ).
  - Requires all three sides to evaluate all six functions.
- Remember to rationalize the denominator when necessary.

2.2 Right Triangle Trigonometry

- Find function values for 30°(, 45°(, and 60° (
  - Start studying the outside of the Unit Circle for sine and cosine of common angles.
  - If you must find the reciprocal, do so before rationalizing the denominator.

2.2 Right Triangle Trigonometry

θ	30° = $\left(\frac{\pi}{6}\right)$,	45° = $\left(\frac{\pi}{4}\right)$	60° = $\left(\frac{\pi}{3}\right)$
sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

2.3 Trigonometric Identities

- Objective:
  - I will be able to use trigonometric identities to find the values of the six main trigonometric functions for an angle.
- Vocabulary:

Trigonometric Identities	Reciprocal identities	Quotient Identities	Pythagorean Identities	Complements
Cofunctions				

2.3 Trigonometric Identities

- Recognize and Use Fundamental Identities
  - Trigonometric identities are relationships between trigonometric functions
  - Identities can be used to find related trigonometric functions.

2.3 Trigonometric Identities

- Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

2.3 Trigonometric Identities

- Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

- Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

2.3 Trigonometric Identities

- Use equal cofunctions of complements
  - Two positive angles are complements if their sum is 90° or .
  - Cofunctions:
    - Any pair of trigonometric functions and for which

$$f(\theta) = g(90° - \theta) \text{ and } g(\theta) = f(90° - \theta)$$

2.3 Trigonometric Identities

- Cofunction Identities

$\sin\theta = \cos(90° - \theta)$ $\cos\theta = \sin(90° - \theta)$

$\tan\theta = \cot(90° - \theta)$ $\cot\theta = \tan(90° - \theta)$

$\sec\theta = \csc(90° - \theta)$ $\csc\theta = \sec(90° - \theta)$

If θ is in radians, replace *90°* with *π/2*

2.4 Applications of Trigonometry

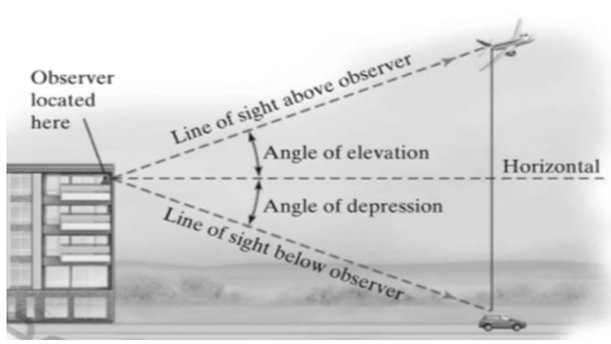
- Objective:
  - I will understand elevation and depression, be able to identify angles of elevation and depression. I will be able to apply these concepts to real-world applications.
- Vocabulary:

Angle of elevation	Angle of depression		

2.4 Applications of Trigonometry

- Use right triangle trigonometry to solve applied problems
  - Many applications of right triangle trigonometry involve the angle made with an imaginary horizontal line.
    - The angle above the horizontal is called the angle of elevation.
    - The angle below the horizontal is called the angle of depression.

2.4 Applications of Trigonometry



- Note: the vertex of the angle can be to the right or to the left.

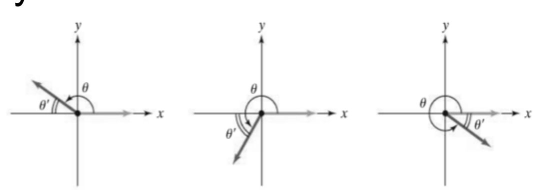
2.5 Reference Angles

- Objective:
  - I will be able to find a reference angle for obtuse angles. I will be able to use the reference angle to evaluate trigonometric functions of obtuse angles.
- Vocabulary:

Reference angles			

2.5 Reference Angles

- Find Reference Angles
  - Trigonometric functions of positive obtuse angles and all negative angles, are evaluated by making use of a positive acute angle.
  - Reference angle – the positive acute angle θ' formed by the terminal side of θ and the x-axis.

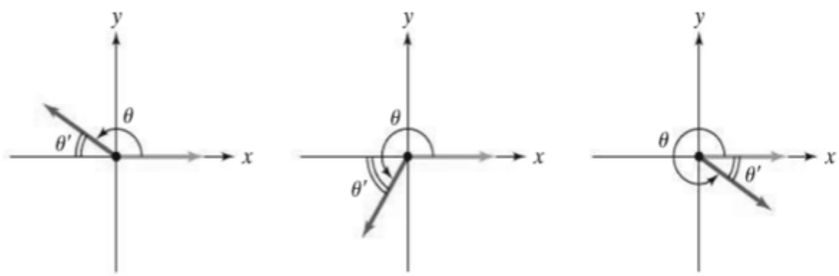


If $90° < \theta < 180°$, then $\theta' = 180° - \theta$.

If $180° < \theta < 270°$, then $\theta' = \theta - 180°$.

If $270° < \theta < 360°$, then $\theta' = 360° - \theta$.

2.5 Reference Angles



If 90° < θ < 180°, then θ′ = 180° − θ.

If 180° < θ < 270°, then θ′ = θ − 180°.

If 270° < θ < 360°, then θ′ = 360° − θ.

2.5 Reference Angles

- Finding reference angles for angles greater than 360° (2π) or less than –360° (–2π)
  - Find a positive angle α less than 360° or 2π that is coterminal with the given angle.
  - Draw α in standard position.
  - Use the drawing to find the reference angle for the given angle. The positive acute angle formed by the terminal side of α and the x-axis is the reference angle.

2.5 Reference Angles

- Use reference angles to evaluate trigonometric functions
  - The values of the trigonometric functions of a given angle, θ, are the same as the values of the trigonometric functions of the reference angle, θ', except possibly for the sign

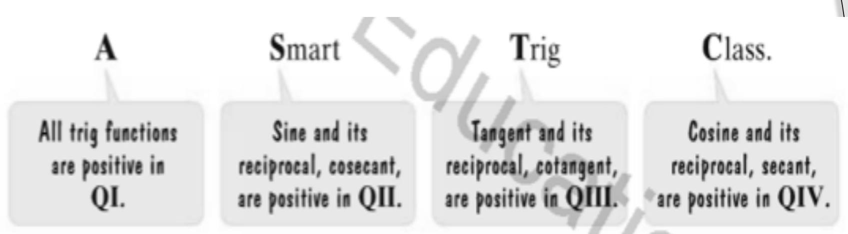
2.5 Reference Angles

- Procedure for using reference angles to evaluate trigonometric functions:
  - Find the associated reference angle, θ', and the function value for θ'.
  - Use the quadrant in which θ lies to prefix the appropriate sign to the function value in the first step.

2.5 Reference Angles

- Use the signs of the trigonometric functions
  - If θ is not a quadrantal angle, the sign of a trigonometric function depends on the quadrant in which θ lies.
    - Quadrant I: all functions are positive
    - Quadrant II: sine and cosecant are positive
    - Quadrant III: tangent and cotangent are positive
    - Quadrant IV: cosine and secant are positive

2.5 Reference Angles



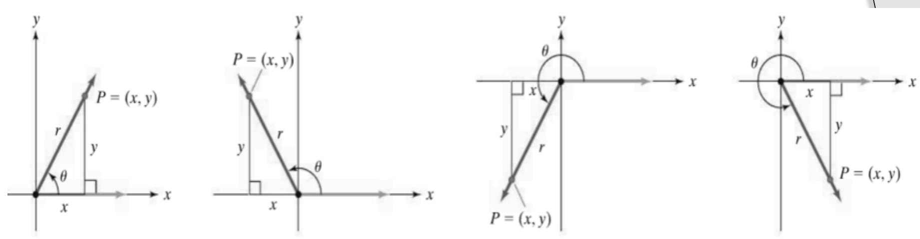
2.6 Trigonometric Functions

- Objective:
  - I will be able to use the definition of trigonometric functions of any angle to evaluate trig functions for any angle. I will be able to determine the sign of a trig function given the quadrant in which lies the terminal side of the angle.

2.6 Trigonometric Functions

- Use the definition of trigonometric functions of any angle
  - In the examples below, the angles are in standard position, and P(x, y) is r units from the origin on the terminal side of θ.
  - A right triangle is formed by drawing the line segment from P perpendicular to the x-axis. In this case, y is opposite θ, and x is adjacent to θ.

2.6 Trigonometric Functions



(a) θ lies in quadrant I. (b) θ lies in quadrant II. (c) θ lies in quadrant III. (d) θ lies in quadrant IV.

2.6 Trigonometric Functions

- Definitions of Trigonometric Functions of Any Angle
  - P (x,y) is any point on the terminal side other than the origin.
  - $r = \sqrt{(x + y)}$ (the distance from the origin to P)
  - The six trigonometric ratios of θ are defined by the following ratios:

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y};\ y \neq 0$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x};\ x \neq 0$$

$$\tan\theta = \frac{y}{x};\ x \neq 0 \qquad \cot\theta = \frac{x}{y};\ y \neq 0$$