Unit 3
Trig II

AFM
Mrs. Valentine

3.1 Trig and Periodic Functions

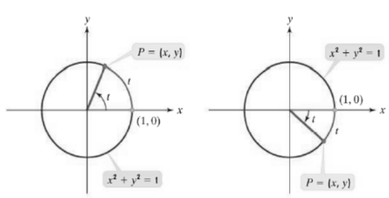
- *Obj.*: I will be able to use a unit circle to find values of sine, cosine, and tangent. I will be able to find the domain and range of sine and cosine. I will understand even and odd trigonometric functions and be able to use their periodicity.
- Vocabulary

Unit Circle	Circular Functions	Even Functions	Odd Functions	Periodic Function
Period				

3.1 Trig and Periodic Functions

- Unit Circle
  - A unit circle is a circle of radius 1, with its center at the origin of a rectangular coordinate system.
  - Equation: $x^2 + y^2 = 1$
  - In a unit circle, the radian measure of the central angle is equal to the length of the intercepted arc.
  - If the angle, t, is a real number in radians, then for each t, there corresponds a point *P(x,y)* on the unit circle.

3.1 Trig and Periodic Functions



(a) t is positive. (b) t is negative.

$\sin t = y$ $\quad$ $\csc t = \frac{1}{y}; y \neq 0$

$\cos t = x$ $\quad$ $\sec t = \frac{1}{x}; x \neq 0$

$\tan t = \frac{y}{x}; x \neq 0$ $\quad$ $\cot t = \frac{x}{y}; y \neq 0$

- Because this definition expresses function values in terms of coordinates of a point on a unit circle, the trigonometric functions are sometimes called circular functions.

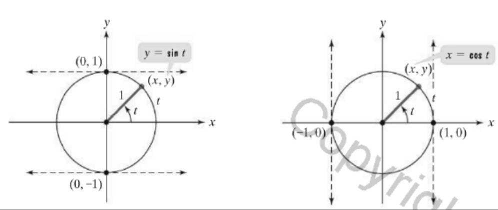
3.1 Trig and Periodic Functions

- Domain and Range of Sine and Cosine Functions
  - The value of a trigonometric function at the real number t is its value at an angle of t radians.
  - The domains and ranges of each trigonometric function can be found from the unit circle definition:

$$\sin t = y \quad \text{and} \quad \cos t = x$$

3.1 Trig and Periodic Functions

- Sine:
  - t can be any real number, so the domain is *(–∞,∞)*
  - because the radius of the unit circle is 1, the range is *[–1,1]*
- Cosine:
  - t can be any real number, so the domain is *(–∞,∞)*
  - because the radius of the unit circle is 1, the range is *[–1,1]*

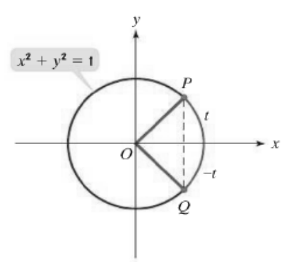


3.1 Trig and Periodic Functions

- Even and Odd Trigonometric Functions
  - A function is even if $f(-t) = f(t)$ and odd if $f(-t) = -f(t)$.
  - The cosine function is an even function
  - The sine function is an odd function.

3.1 Trig and Periodic Functions

- On the figure to the right:
  - P:($\cos t$, $\sin t$)
  - Q:($\cos(-t)$, $\sin(-t)$)
  - The x-coordinates of P and Q are the same, thus $\cos(-t) = \cos t$
  - The y-coordinates of P and Q are negatives of each other, thus $\sin(-t) = -\sin t$



3.1 Trig and Periodic Functions

- Cosine and secant functions are even

$\cos(-t) = \cos t$ $\sec(-t) = \sec t$

- Sine, cosecant, tangent, and cotangent functions are odd.

$\sin(-t) = -\sin t$ $\csc(-t) = -\csc t$

$\tan(-t) = -\tan t$ $\cot(-t) = -\cot t$

3.1 Trig and Periodic Functions

- Periodic Functions
  - Certain patterns in nature repeat again and again. A behavior is periodic if the repetition continues infinitely.
  - A period is the time it takes to complete one full cycle.
  - A function *f* is periodic if there exists a positive number *p* such that

$$f(t+p) = f(t)$$

for all t in the domain of *f*. The smallest positive number *p* for which *f* is periodic is called the period of *f*.

3.1 Trig and Periodic Functions

- Sine and cosine functions are periodic and have period 2π:

$\sin(t+2\pi) = \sin t$ and $\cos(t+2\pi) = \cos t$

- Tangent and cotangent functions are periodic and have period π:

$\tan(t+\pi) = \tan t$ and $\cot(t+\pi) = \cot t$

3.1 Trig and Periodic Functions

- Repetitive behavior of sine, cosine, and tangent functions:

$$\sin(t+2\pi n)=\sin t$$

$$\cos(t+2\pi n) = \cos t$$

$$\tan(t+\pi n) = \tan t$$

where n is the number of full cycles.

3.2 Graphs of Sine

- *Obj.*: I will be able to graph sine functions and the variations. I will be able to determine amplitude, period, and phase shift for a sine graph.

- Vocabulary

Sine Curve	Amplitude	Phase Shift		

3.2 Graphs of Sine

- The Graph of y = sin x
  - The trigonometric functions can be graphed in a rectangular coordinate system by plotting points whose coordinates satisfy the function.
  - In this case, we graph *y* = sin *x* by listing some points on the graph.
    - Graph the function on the interval *[0,2*π*]* because the period is *2*π.
    - The rest of the graph is made up of repetitions of this portion.

3.2 Graphs of Sine

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0

As x increases from 0 to $\frac{\pi}{2}$, y increases from 0 to 1.

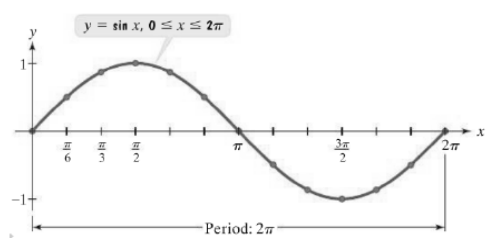
As x increases from $\frac{\pi}{2}$ to π, y decreases from 1 to 0.

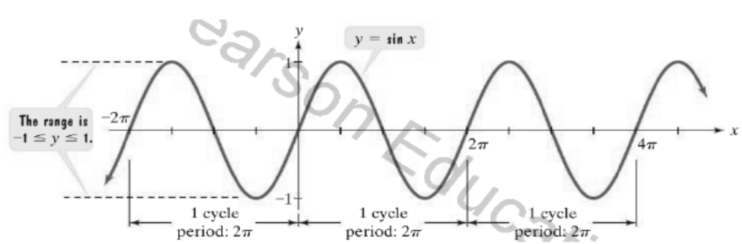
As x increases from π to $\frac{3\pi}{2}$, y decreases from 0 to −1.

As x increases from $\frac{3\pi}{2}$ to 2π, y increases from −1 to 0.

- Approximate √*(3)/2≈0.87* in order to plot it.
- Mark off the x-axis in terms of π to avoid rounding the x-values.
- Connect the points with a smooth curve

3.2 Graphs of Sine





3.2 Graphs of Sine

- The graph of $y = \sin x$ will have the following properties:
  - Domain: $(-\infty,\infty)$ and Range: $[-1,1]$
  - Period: 2π
  - Odd function: $\sin(-x) = -\sin x$

3.2 Graphs of Sine

- Graphing Variations of y = sin x
  - One complete cycle of the sine curve includes three x-intercepts, one maximum point, and one minimum point.
  - The key points on the graph can be found using the equations below to find the x-values, and evaluate the function at each x to find the y-values.

x_1 = value of x where the cycle begins

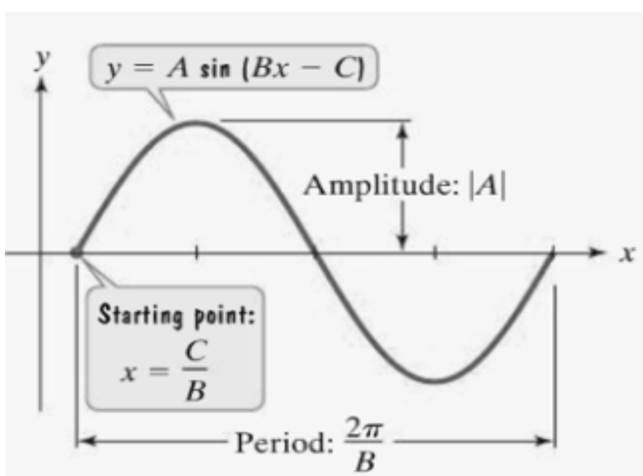
$$x_2 = x_1 + \frac{\text{period}}{4}$$
$$x_3 = x_2 + \frac{\text{period}}{4}$$
$$x_4 = x_3 + \frac{\text{period}}{4}$$
$$x_5 = x_4 + \frac{\text{period}}{4}.$$

Add "quarter-periods" to find successive values of x.

3.2 Graphs of Sine

- Graphing $y = A \sin(Bx - C)$, $B > 0$
  - Amplitude
    - Maximum height of the graph
    - Equal to $|A|$
  - Period $= 2\pi/B$
  - Phase Shift
    - The horizontal shift to the sine graph
    - New starting point: C/B
    - One period is $C/B \le x \le (C/B + 2\pi/B)$

3.2 Graphs of Sine



3.2 Graphs of Sine

- Steps for graphing:
  - Identify the period, amplitude, and phase shift.
  - Find the x-values for the 5 key points, starting at the beginning of the cycle.
  - Find the y-values for the 5 key points by evaluating the function at each value of x from the second step.
  - Connect the five key points with a smooth curve and graph one complete cycle.
  - Extend the graph to the left or right as needed.

3.3 Graphs of Cosine

- *Obj.*: I will be able to graph cosine functions and the variations. I will be able to determine amplitude, period, and phase shift for a cosine graph.

- Vocabulary

Sinusoidal Graphs			

3.3 Graphs of Cosine

- The Graph of y = cos x
  - Graph *y* = cos *x* by listing some points on the graph.
    - Graph the function on the interval *[0,2*π*]* because the period is *2*π.
    - The rest of the graph is made up of repetitions of this portion.

3.3 Graphs of Cosine

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1

As x increases from 0 to $\frac{\pi}{2}$, y decreases from 1 to 0.

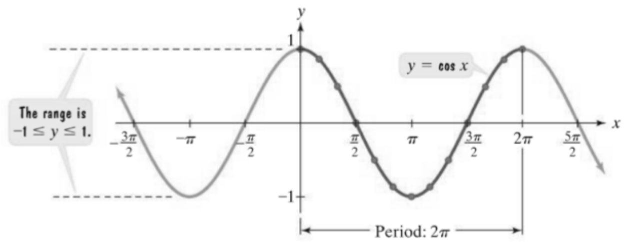
As x increases from $\frac{\pi}{2}$ to π, y decreases from 0 to −1.

As x increases from π to $\frac{3\pi}{2}$, y increases from −1 to 0.

As x increases from $\frac{3\pi}{2}$ to 2π, y increases from 0 to 1.

- Approximate √*(3)/2 ≈ 0.87* in order to plot it.
- Mark off the x-axis in terms of π to avoid rounding the x-values.
- Connect the points with a smooth curve.

3.3 Graphs of Cosine



- The graph of $y = \cos x$ will have the following properties:
  - Domain: $(-\infty, \infty)$ and Range: $[-1, 1]$
  - Period: 2π
  - Even function: $\cos(-x) = \cos x$

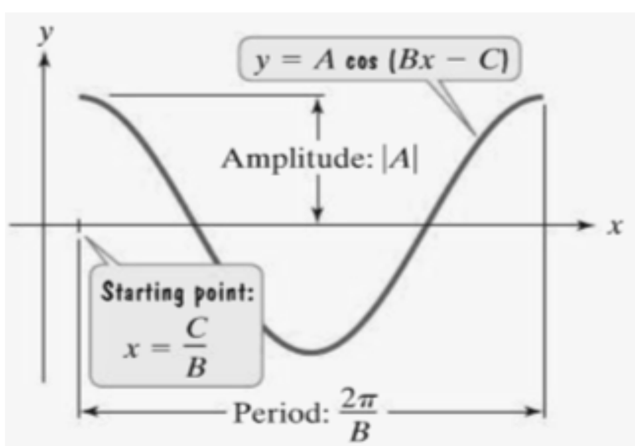
3.3 Graphs of Cosine

- The graph of $y = \cos x$ is the graph of $y = \sin x$ with a phase shift of $-\pi/2$.
  - Tracing the cosine graph from $x = -\pi/2$ to $x = 3\pi/2$ is one complete cycle of the sine curve.
  - Identity: $\cos x = \sin(x + \pi/2)$
  - Because of the similarity, the graphs of sine functions and cosine functions are called sinusoidal graphs.

3.3 Graphs of Cosine

- Graphing Variations of y = cos x
  - Graphing $y = A \cos (Bx – C)$, $B>0$
    - Amplitude: $|A|$
    - Period: $2\pi/B$
    - Phase Shift: C/B
      (One period is $C/B \le x \le C/B + 2\pi/B$)
- Steps for graphing are the same as for graphs of sine functions.

3.3 Graphs of Cosine



3.4 Vertical Shifts and Periodic Models

- Obj.: I will be able to translate the graph of a sine and/or cosine function vertically. I will be able to apply graphing sine and cosine functions to modeling data.

3.4 Vertical Shifts and Periodic Models

- Vertical Shifts of Sinusoidal Graphs
  - Forms:
    $y = A \sin (Bx - C) + D$ and $y = A \cos (Bx - C) + D$
  - Constant D causes a vertical shift in the graph.
    - If D is positive, the graph shifts upward D units
    - If D is negative, the graph shifts downward $|D|$ units

3.4 Vertical Shifts and Periodic Models

- Oscillations are now around line $y=D$
  - Maximum value of y is $D+|A|$
  - Minimum value of y is $D-|A|$
- Modeling Periodic Behavior
  - Periodic data can be better understood by graphing.

3.4 Vertical Shifts and Periodic Models

- Additionally, some graphing calculators have a SINe REGression feature.
  - Minimum of four points required
  - Gives the function in the form $y = A \sin (Bx-C)+D$ of the best fit for wavelike data.

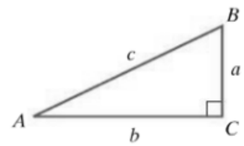
3.5 Applications

- Obj.:I will be able to solve right triangles. I will be able to apply trigonometric functions to trigonometric bearings and simple harmonic motion.
- Vocabulary

Solving a right triangle	Bearing	Simple Harmonic Motion	Equilibrium Position	Frequency

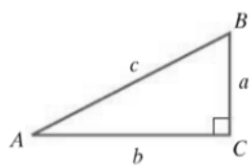
3.5 Applications

- Solve Right Triangles
  - Solving a right triangle means finding the missing lengths of its sides and the measurements of its angles.
  - In addition to the right angle, you will need at least two additional pieces of information to solve a right triangle.



3.5 Applications

- The sum of the two acute angles is 90°.
- Sine, cosine, and/or tangent can be used to find any missing sides.
- Recall that if you know two sides of a right triangle but not either of the acute angles, you can use the inverse of a trigonometric function to solve for the angle.



3.5 Applications

- Trigonometry and Bearings
  - In navigation and surveying problems, the word bearing is used to specify the location of one point relative to another.
  - The bearing from point O to point P is the acute angle, measured in degrees, between ray OP and a north-south line.

3.5 Applications

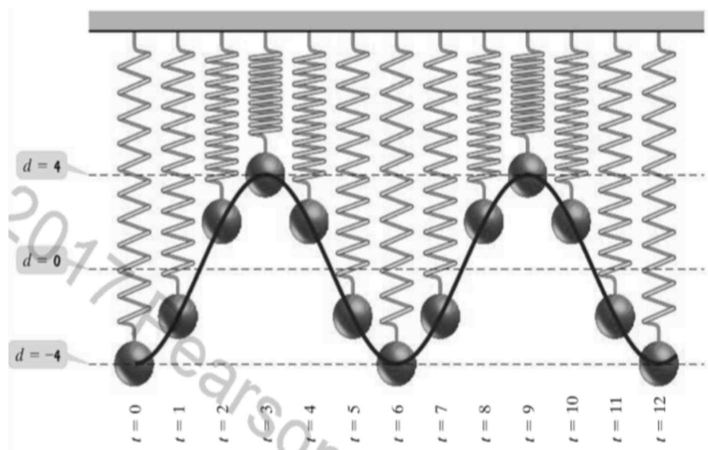
- Each bearing has three parts: a letter (N or S), the measure of an acute angle, and a letter (E or W).

1. Start with the north/south side (N if the north side of the north-south line, S if the south side of the north-south line)
2. Write the measure of the acute angle.
3. If the acute angle is measured on the east side of the north-south line, write E last. If on the west side, write W last.

3.5 Applications

- Simple Harmonic Motion
  - Trigonometric functions are used to model vibratory or oscillatory motion (such as the motion of a vibrating guitar string, the swinging of a pendulum, or the bobbing of an object attached to a spring).
    - Consistent oscillations are called simple harmonic motion.
    - The rest position is called the equilibrium position (horizontal line that runs through the center of the graph).

3.5 Applications



3.5 Applications

- An object that moves on a coordinate axis is in simple harmonic motion if its distance from the origin, d, at the t is given by either

  $d = a\cos(\omega t)$ or $d = a\sin(\omega t)$

  - Amplitude: $|a|$ (maximum displacement)
  - Period: $2\pi/\omega$, $\omega>0$ (time for one complete cycle)

3.5 Applications

- Frequency: $f = \omega / 2\pi$, $\omega > 0$
  - Describes the number of complete cycles per unit time.
  - Reciprocal of the period: $f = 1/period$