UNIT 3
QUADRATICS II

M2 12.1-8, M2 12.10, M1 4.4

3.1 Quadratic Graphs

- Objective
  - I will be able to identify quadratic functions and their vertices, graph them and adjust the height and width of the parabolas.

- Vocabulary

• Standard Form of a Quadratic Function			• Parabola
• Axis of Symmetry	• Vertex	• Minimum	• Maximum
• Quadratic Function		• Quadratic Parent Function	

3.1 Quadratic Graphs

- Quadratic Functions
  - Nonlinear (rate of change is not constant)
  - Standard Form of Quadratic Function: $f(x) = ax^2 + bx + c$ where $a \neq 0$
  - Quadratic Parent Function: $f(x) = x^2$ or $y = x^2$ (simplest quadratic)

3.1 Quadratic Graphs

- Identifying a Vertex
  - The graph of a quadratic function is called a parabola
    - U-shaped curve
    - Symmetrical (has an axis of symmetry)
    - Has a maximum (opens down) or minimum (opens up)

3.1 Quadratic Graphs

- Highest or lowest point of a parabola is the vertex (h, k)
  - Coordinate point
  - On the axis of symmetry
  - In $y=ax^2+bx+c$, domain is $\mathbb{R}$ and

a > 0 (positive)	a < 0 (negative)
minimum	maximum
Range: $y \geq k$	Range: $y \leq k$

3.1 Quadratic Graphs

- Graphing Parabolas
  - When $y=ax^2$
    - Use symmetry to graph quickly
    - Find the coordinates of the vertex and a couple points on one side of the vertex. Reflect the points across the axis of symmetry.
    - Axis of symmetry is the y-axis (x = 0)
    - Vertex is the origin (0, 0)

3.1 Quadratic Graphs

- When $y = ax^2 + c$
  - The y-axis is the axis of symmetry in this form.
  - The value of "c" translates the graph up or down → vertex is (0, c)
- Comparing Widths of Parabolas
  - The lead coefficient (a) affects the width of a parabola.
  - When $|m| < |n|$, the graph of $y = mx^2$ is wider than the graph of $y = nx^2$.

3.1 Quadratic Graphs

- Falling Object Model
  - As an object falls, speed continues to increase as height decreases
  - Ignoring air resistance, the object's height can be modelled with the function $h = -16t^2 + c$
    - Height (h) is in feet
    - Time (t) is in seconds
    - Initial height (c) is in feet

3.2 Quadratic Functions

- Objective
  - I will be able to graph and analyze quadratic functions in the form y = $ax^2 + bx + c$. I will be able to find a parabola that fits through any three nonlinear points.
- Vocabulary
  - None

3.2 Quadratic Functions

- Graphing $y=ax^2+bx+c$
  - Y-intercept: c
  - Axis of Symmetry: $x=b/-2a$
  - Vertex (h, k)
    - On the axis of symmetry
    - Finding vertex:
      - $h=b/-2a$
      - $k=ah^2+bh+c$

3.2 Quadratic Functions

- Find the vertex, the y-intercept, and a few other points, then plot the graph.
- Check:
  - Does the parabola open the right way?
  - Is the parabola symmetrical?
  - Is the vertex on the axis of symmetry?

3.2 Quadratic Functions

- Using the Vertical Motion Model
  - If an object is projected into the air with an initial upward velocity, v, an initial height, c, and time, t, continues with no additional force acting on it, the formula

    $h=-16t^2+vt+c$.

3.2 Quadratic Functions

- Writing an Equation of a Parabola
  - You can use the standard form of the quadratic function and any three points of the parabola to find an equation.
  - If three points are nonlinear, no two of which are in line vertically, then they are on the graph of exactly one quadratic function.
  - This method uses a system of linear equations to find a, b, and c.

3.2 Quadratic Functions

- Using Quadratic Regression
  - Quadratic regression is a process used to find the equation of a parabola that "best fits" a set of coordinates.
    - Best to use three or more nonlinear points.

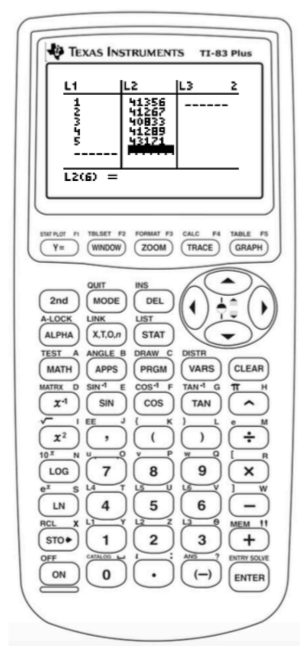
3.2 Quadratic Functions

- Need the following steps in the graphing calculator:
  - Press the “STAT” button
  - Select “1: Edit”
  - Enter your x-values in L1
  - Enter your y-values in L2
  - Press the “STAT” button again.
  - At the top of the screen, scroll to “CALC”
  - Select “5:QuadReg”
- The resulting screen will give you the a, b, and c values of the quadratic equation in standard form.

3.2 Quadratic Functions

- QuadReg
  $y=ax^2+bx+c$
  a=345.1428571
  b=-1705.657143
  c=42903.6
  R^2=.903486496

- R^2 indicates how well the parabola fits. The closer to “1,” the better the fit.



3.3 Solving Quadratic Equations

- Objective
  - I will be able to solve quadratic equations using graphing and/or square roots. I will be able to evaluate whether a solution is reasonable given an application of quadratics.
  - Vocabulary

• Zero of a Function	• Root of an Equation
• Quadratic Equation	• Standard Form of a Quadratic Equation

3.3 Solving Quadratic Equations

- Solving by Graphing
  - A quadratic equation is an equation written in the form $ax^2+bx+c=0$ where $a\neq 0$.
  - This is the standard form of a quadratic equation.
  - Solutions to a quadratic equation are the x-intercepts of its related parabola.
    - Often called roots of the equation or zeros of the function.
    - Quadratics can have 0, 1, or 2 real solutions.
    - May be real or imaginary. For now, we'll focus on real solutions.

3.3 Solving Quadratic Equations

- Solving Using Square Roots
  - Equations in the form $x^2 = k$ can be solved by finding the square root of each side.
  - Remember that when taking a square root, you get both a positive and negative value. (Ex: For $x^2 = 81$, the solutions are $\pm\sqrt{81} = \pm 9$.)
  - Rearrange the equation for the x^2 term BEFORE taking square roots.
  - Works only when b = 0 or in completed square form.

3.3 Solving Quadratic Equations

- Choosing a Reasonable Solution
  - In many cases when solving real-world problems modeled by quadratic equations, the negative square root may not be a reasonable solution.
    - Finding length
    - Finding time elapsed
    - Finding mass, etc.

3.4 Factoring to Solve

◦Objective
- ◦ I will be able to use factoring and the zero product property to solve quadratic equations. I will be able to use the factored form of a quadratic equation to graph its parabola.

◦Vocabulary

• Zero Product Property	

3.4 Factoring to Solve

◦Zero-Product Property
- ◦Multiplication Property of Zero: for any real number, a, $a*0=0$
- ◦Zero Product Property: for any real numbers a and b, if $ab=0$, then $a=0$ or $b=0$.
- ◦Quadratic equations written in factored form can be solved using the zero product property.

3.4 Factoring to Solve

- Solving by Factoring
  - Once a quadratic equation is in standard form ($ax^2+bx+c=0$), factor the quadratic to put it in factored form. You may have to add or subtract terms to both sides to put it in standard form.

3.4 Factoring to Solve

- GCF – all quadratic equations, usually followed by another method
- Reverse FOIL
- Tic-Tac-Toe Method

  } 3 terms, not perfect squares

- Difference of Squares – subtraction of two perfect squares
- Perfect Square Trinomial – 3 terms, perfect squares
- Grouping – 4 terms with common binomial factor.
- Use the zero product property to solve the equation.

3.4 Factoring to Solve

- Using Factored Form to Graph a Function
  - You can find the zeros of the function from factored form.
  - The axis of symmetry is in the middle of the zeros, so you can average them together to find the axis of symmetry (x = h).
  - Calculate $f(h)$.
  - Plot the three points and draw the parabola through them.

3.6 Completing the Square

- Objective
  - I will be able to find values to complete the square, find the vertex using the completed square form, and solve quadratic equations using after the equation has been converted.

- Vocabulary

• Completing the Square	

3.5 Completing the Square

- Finding c to Complete the Square
  - You can change the expression $x^2 + bx$ into a perfect-square trinomial by adding $(b/2)^2$ to $x^2 + bx$.
  - This is called completing the square.
  - The process is the same whether b is positive or negative.

3.5 Completing the Square

- Solving $x^2 + bx + c = 0$
  - Start by subtracting c from both sides of the equation.
  - Add $(b/2)^2$ to both sides of the equation.
  - Factor the new perfect-square trinomial.
  - Take the square root of both sides.
  - Write as two equations (taking the square root gives both a positive and negative).
  - Solve for x in each equation.

3.5 Completing the Square

◦Finding the Vertex by Completing the Square

◦The equation $y=a(x-h)^2+k$ is called the completed square form, or vertex form, of a quadratic equation.

◦The vertex can be found by changing the sign of the value in the parentheses for h and keeping k as is. (h, k)

◦Completing the Square when $a\neq 1$

◦Divide all terms by a.

◦Solve by completing the square using the same procedure from this point forward.

3.6 Quadratic Formula & Discriminant

◦Objective

◦I will be able to use the quadratic formula to find solutions to quadratic equations. I will be able to select the best method for solving quadratic equations and use the discriminant to predict how many real solutions a quadratic equation will have.

◦Vocabulary

• Quadratic Formula	• Discriminant

3.6 Quadratic Formula & Discriminant

- Quadratic Formula
  - A quadratic equation can have two, one, or no real-number solutions (never more than two).
  - Any quadratic equation in standard form ($ax↑2 +bx+c=0, a≠0$) can be solved using the quadratic formula:

$x=-b±√b↑2 -4ac /2a$

  - When the radicand in the quadratic formula is not a perfect square, you can use a calculator to approximate (round) the solutions of an equation.

3.6 Quadratic Formula & Discriminant

- Choosing an Appropriate Method

Method	**When to Use**
Graphing	If you have a graphing calculator handy
Square Roots	If the equation has no x-term
Factoring	If you can factor the equation easily
Completing the Square	If the coefficient of x^2 is 1, but cannot easily factor
Quadratic Formula	If the equation cannot be factored easily or at all.

3.6 Quadratic Formula & Discriminant

- Using the Discriminant
  - The discriminant is the radicand of the quadratic formula: $b^2 - 4ac$
  - Can be used to determine how many real solutions a quadratic equation has
    - If positive: 2 real solutions
    - If zero: 1 real solution
    - If negative: 0 real solutions

3.7 Complex Numbers

- Objective
  - I will be able to simplify the square root of a negative number, plot complex numbers in the complex plane, and find the absolute value of a complex number. I will be able to add, subtract, multiply, and divide complex numbers.

- Vocabulary

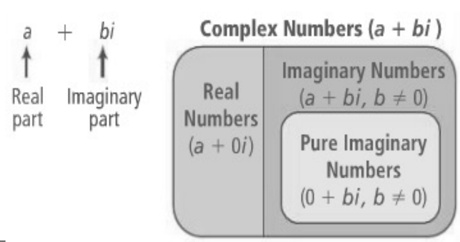
• Imaginary Unit	• Imaginary Number	• Complex Number
• Complex Number Plane	• Pure Imaginary Number	
• Absolute Value of a Complex Number	• Complex Conjugates	

3.7 Complex Numbers

- Simplifying a Number Using *i*
  - Complex numbers are based on a number whose square is –1
  - The imaginary unit (*i*)
    - the complex number whose square is –1 ($i^2 = -1$)
    - $i = \sqrt{-1}$
  - For any positive number, a, $\sqrt{-a} = \sqrt{-1 * a} = \sqrt{-1} * \sqrt{a} = i\sqrt{a}$

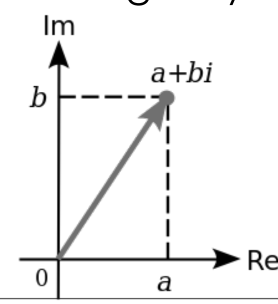
3.7 Complex Numbers

- Graphing in the Complex Number Plane
  - Imaginary number: any number in the form $a+bi$ where *a* and *b* are real and $b \neq 0$.
    - If $b=0$, the number is real.
    - Pure imaginary number: $a=0$ and $b \neq 0$



3.7 Complex Numbers

- Complex Number Plane
  - Just like the real coordinate plane.
  - Point (a,b) represents $a+bi$
  - Real numbers are on the horizontal axis while imaginary part is on the vertical axis.
- Absolute Value of a Complex Number
  - The distance from the origin to the point in the complex plane
  - $|a+bi|=\sqrt{a^2+b^2}$



3.7 Complex Numbers

- Adding and Subtracting Complex Numbers
  - Add/subtract like terms (real numbers with real numbers, imaginary numbers with imaginary numbers).
  - If the sum of two complex numbers is 0 or $0+0i$, each number is the additive inverse of the other.
- Multiplying Complex Numbers
  - Multiply complex numbers $a+bi$ and $c+di$ like any other binomials.
    - $(bi)(di)=bdi^2=-bd$

3.7 Complex Numbers

◦Dividing Complex Numbers

◦ $a+bi$ and $a-bi$ are complex conjugates because they multiply to a real number.

◦Multiply the numerator and denominator by the complex conjugate of the denominator and simplify.

◦Finding Imaginary Solutions

◦Every quadratic has exactly two complex number solutions (that sometimes are real numbers).

3.8 Systems of Equations

◦Objective

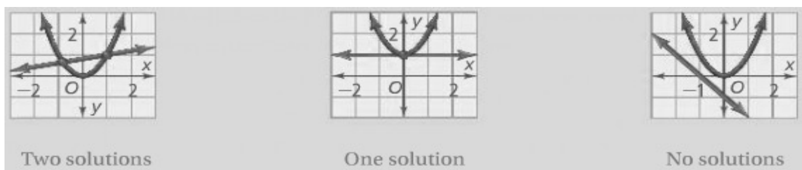
◦I will be able to solve a system of equations involving quadratic equations using graphing, elimination, and substitution.

◦Vocabulary

• Substitution Method	• Elimination Method
• Solution of a System of Equations	

3.8 Systems of Equations

◦Solving by Graphing
  ◦Solution of a system of equations
    ◦Coordinates that make all equations true
    ◦Written as coordinate pairs
    ◦Intersection points of graphs
  ◦Systems of linear and quadratic equations



3.8 Systems of Equations

◦Solving by Elimination
  ◦Add or subtract the equations to cancel out a variable.
  ◦Solve the remaining equation for the left over variable.
  ◦Substitute the known variable back into one equation to find the second.
  ◦Write answers as coordinate pairs.

3.8 Systems of Equations

- Solving by Substitution
  - Replace one variable with an equivalent expression containing the other variable.
  - Easiest if you solve both equations for the same variable, and then set the two equations equal to each other.

3.8 Systems of Equations

- Solving with a Graphing Calculator
  - Enter the equations on the Y= screen.
  - Press “graph” to display the system
  - Use the “CALC” feature. Select “INTERSECT” and place the cursor close to a point of intersection. Press “enter” 3 times.
  - Repeat the steps for each intersection.