UNIT 4:
RATIONAL AND RADICAL EXPRESSIONS

M1 5.8, M2 10.1-4, M3 5.4-5, 6.5,8

4.1 Product Rule

- Objective
  - I will be able to multiply powers when they have the same base, including simplifying algebraic expressions and scientific notation. I will be able to evaluate expressions with rational exponents.

- Vocabulary

○ Base	○ Scientific Notation

4.1 Product Rule

- Multiplying Powers
  - When multiplying powers with the same base, add the exponents.
    - Base – number raised to an exponent
    - Exponent – a number that tells how many times the base should be multiplied by itself.
  - The rule:
    - $a^m * a^n = a^{m+n}$ where $a \neq 0$ and m & n are rational
    - $a^m * b^n = a^m b^n$ (cannot combine exponents with different bases)

4.1 Product Rule

- … in Algebraic Expressions
  - When multiplying algebraic terms together, multiply the coefficients and add the exponents.
  - THE BASE STAYS THE SAME
  - Don't forget to add the assumed "1" if no exponent is listed (Ex: $d^2 * d = d^3$)

4.1 Product Rule

- … with Scientific Notation
  - Scientific Notation –any number written as $a \times 10^b$, where $1 \leq a < 10$.
    - a is the coefficient, 10 is the base.
    - If you move the decimal to the right, subtract from the exponent. If you move the decimal to the left, add to the exponent.
      - Ex: 256000 is written as 2.56×10^5
      - Ex: 0.000345 is written as 3.45×10^{-4}

4.1 Product Rule

- Can be multiplied just like algebraic expressions with the same base.
  - In your calculator, 4.567×10^{14} appears as *4.567E14*.
  - It is okay to leave negative exponents in scientific notation.

4.1 Product Rule

- Simplifying Expressions with Rational Exponents
  - Fractional exponents are called rational exponents.
  - If $a^{1/m}=b$, then $a=b^m$
  - You can use this (or your calculator) to simplify expressions with rational exponents.
    - If you use your calculator, you MUST put the rational exponent in parentheses!!

4.1 Product Rule

- Additional Rules to Remember
  - Zero as an Exponent: $a^0=1$ where $a\neq 0$
  - Negative Exponents Property: $a^{-n}=1/a^n$ where $a\neq 0$ and n is rational

4.2 Power Rule

- Objective
  - I will be able to raise powers to powers, raise products to powers, and apply these rules to scientific notation.
- Vocabulary
  - None

4.2 Power Rule

- An item raised to a power may be raised to a second power by multiplying the powers.
  - The base stays the same
  - $(x^m)^n = x^{mn}$
- If a monomial in parentheses is raised to a power, that power must be applied to each factor in the parentheses.
  - $(xy)^n = x^n y^n$

4.2 Power Rule

- If a polynomial is parentheses is raised to a power, rewrite as separate factors and use polynomial multiplication
  - NOTE: Many theorems and procedures exist to speed this process and are covered in units on polynomials. Chief among them is Binomial Theorem and FOIL.
- For numbers in Scientific Notation:
  - Use the distributive property of exponents
  - Put back in scientific notation

4.2 Power Rule

- Remember your order of operations:
  - Parentheses
  - Exponents
  - Multiplication and Division
  - Addition and Subtraction
  - Though not a required step in the order of operations, grouping terms can sometimes make it easier to see what goes together.

4.3 Quotient Rule

- Objective
  - I will be able to divide powers with like bases and take the power of a quotient. I will be able to apply these rules to scientific notation.
- Vocabulary
  - None

4.3 Quotient Rule

- To divide two numbers with the same base, subtract the exponents
  - Rule: $\dfrac{x^m}{x^n} = x^{m-n}$
  - NOTE: For exponents, division is subtraction

4.3 Quotient Rule

- Remember Negative Exponents mean division
  - Rule: $x^{-n} = \frac{1}{x^n}$
  - Note: Simplify a single term with exponents means to make it so that each variable appears only once and that all exponents are positive. *Remember "terms" do not have addition or subtraction.*

4.3 Quotient Rule

- The power of a quotient is the quotient of the powers
  - Rule: $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$
  - Note: be sure to evaluate powers with numerical bases
- Bringing it together
  - Break down the problem, then handle each set of numbers and each variable in turn.
  - Group coefficients (numbers), and group the like bases.

4.3 Quotient Rule

- Division and Scientific Notation
  - Group the numbers and the 10s just like before.
  - Put it back in scientific notation.

4.4 Radicals and Rational Exponents

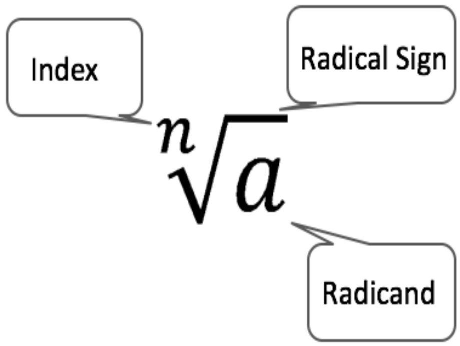
- Objective
  - I will be able to identify parts of a radical expression and convert between rational exponents and radical expressions.

- Vocabulary

o Index	o Radicand

4.4 Radicals and Rational Exponents

- Anatomy of a radical:



- When no index is shown, it is 2: $\sqrt{a} = \sqrt[2]{a}$

4.4 Radicals and Rational Exponents

- For a real number, a, and positive integers m and n,
  - $a^{\frac{1}{n}} = \sqrt[n]{a}$
  - $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

4.4 Radicals and Rational Exponents

- Converting to exponents allows us to simplify radicals that before we could not.
  - Consider $\sqrt[3]{x^2} * \sqrt[5]{x}$:

$$= x^{\frac{2}{3}} * x^{\frac{1}{5}}$$

$$= x^{\frac{13}{15}}$$

$$= \sqrt[15]{x^{13}}$$

4.5 Inverse Variation

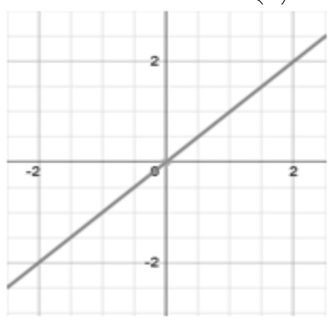
- Objective
  - I will be able to identify direct and inverse variations. I will be able to calculate the constant of variation and set up a model for each variation. I will be able to identify, model, and manipulate combined variations.

- Vocabulary

o Direct Variation	o Inverse Variation
o Asymptote	o Constant of Variation
o Joint Variation	o Combined Variation

4.5 Inverse Variation

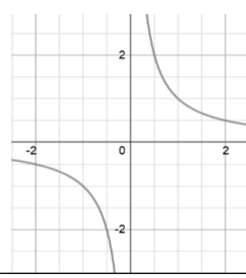
- Direct Variation
  - *y=kx* (a line through the origin)
  - Constant of Variation (k): given a set of data, for each point *y*/*x* is constant. Therefore, *k*=*y*/*x*.



Direct Variation

4.5 Inverse Variation

- Inverse Variation
  - *y=k*/*x*
  - Constant of Variation (k): given a set of data, for each point *xy* is constant. Therefore, *k*=*xy*.
  - Inverse variation graphs have asymptotes (lines the graph approaches but never touches).



4.5 Inverse Variation

- Combined Variation
  - A relation in which one variable varies with respect to each of two or more variables.
  - Ex: *y=kz/x*
  - Joint Variation
    - A direct variation of the product of two or more variables.
    - Ex: *y=kxz*
    - Type of combined variation
    - Constant of Variation (k): given a set of data, for each point *y/xz* is constant. Therefore, *k=y/xz*.

4.6 Transformations of Inverse Variations

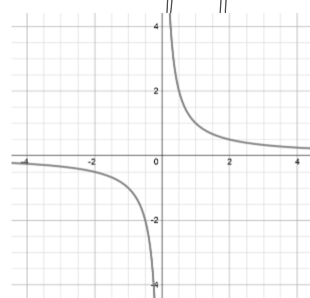
- Objective
  - I will be able to recognize reciprocal functions, analyze how reciprocal functions have been transformed, and sketch reciprocal functions.

- Vocabulary

o Branch	o Reciprocal Function

4.6 Transformations of Inverse Variations

- Reciprocal Function Family
  - Parent function: *y=1/x*
    - Always goes through points (1,1) and (–1, –1)
    - Asymptotes: approaches but never reaches x = 0 and y = 0
    - Domain: *x≠0*
    - Range: *y≠0*



4.6 Transformations of Inverse Variations

- General Equation for a Reciprocal Function:

$$y = \frac{a}{x-h} + k \quad \text{where } a \neq 0$$

  - Always goes through points (h+1, k+a) and (h–1, k–a)
  - Asymptotes: approaches but never reaches x = h and y = k
  - Domain: *x≠h;* Range: *y≠k*
  - Reflections: a < 0 means the graph is flipped over the x-axis
  - Translations: Graph moved right *h* units, and up *k* units (left and down if *h* and *k* are negative).

4.7 Simplifying Radicals

- Objective
  - I will be able to simplify square roots and radicals of other indexes. I will be able to simplify products of radicals with different indexes. I will be able to rationalize the denominator.

- Vocabulary

o Radical Expression	o Rationalize the Denominator

4.7 Simplifying Radicals

- Simplifying Square Roots by Perfect Squares
  - Factor out the largest square you can find.
  - Evaluate that square root, carry the rest.
  - Check for any squares you missed.

4.7 Simplifying Radicals

- The Perfect Squares

x	x^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

x	x^2
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400

x	x^2
21	441
22	484
23	529
24	576
25	625
26	676
27	729
28	784
29	841
30	900

4.7 Simplifying Radicals

- Simplifying Radicals by Prime Factorization
  - Factor into prime factors
  - Group the factors
  - Divide powers of each factor by the index (use remainders)
  - Simplify
  - Recombine terms
- Rational Exponents
  - Rational exponents can be used instead of radicals.
  - This is often useful when dealing with different indexes.

4.7 Simplifying Radicals

- Rationalizing the denominator
  - Radicals cannot be left in the denominator.
  - To rationalize the denominator, multiply the original fraction by the radical in the denominator over itself and simplify.

4.8 Radical Equations

- Objective
  - I will be able to recognize radical equations and solve them. I will be able to identify and eliminate extraneous solutions.

- Vocabulary

o Radical Equation	o Extraneous Solutions	o Radical Function
o Square Root Equation	o Square Root Function	

4.8 Radical Equations

- Radical Equations
  - Equations with the variable in the radicand.
  - Ex: Radical Equations NOT Radical Equations

Radical Equations	NOT Radical Equations
$3 + \sqrt{x} = 10$	$\sqrt{3} + x = 1$
$\sqrt{(x-2)} - 5 = 21$	$\sqrt{9} = x$
$(x+8)^{\frac{2}{3}} = 25$	$2x + \sqrt{16} = 7$

4.8 Radical Equations

- Solving Radical Equations
  - Isolate the radical to one side of the equation
  - Raise both sides of the equation to the same power
  - Simplify
  - Check your Extraneous Solutions
    - Solutions that do not work when you check them.
    - Not part of your answer.
    - If all solutions are extraneous, there is “no solution.”
- NOTE: Squares and square roots are inverses (cube and cube root are inverses, and so on).

4.9 Transformations

- Objective
  - I will be able to identify transformations to radical functions from their equations. I will be able to plot radical functions to find zeros. I will be able to simplify radical functions to find the transformations.
- Vocabulary
  - None

4.9 Transformations

- Square Root Functions
  - Parent Function: $y = \sqrt{x}$
  - General Form: $y = a\sqrt{x - h} + k$
    - a < 0 reflects graph over x-axis
    - a > 1 stretches graph; 0 < a < 1 shrinks graph
    - Translation:
      - Horizontal: h (positive → right, negative → left)
      - Vertical: k (positive → up, negative → down)

4.9 Transformations

- Radical Functions
  - Parent Function: $y = \sqrt[n]{x}$
  - General Form: $y = a\sqrt[n]{x-h} + k$
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      - Vertical: k (positive → up, negative → down)

4.9 Transformations

- Solving Radical Equations by Graphing
  - Plot the graph
  - Use your calculator to find the zeros
    - Y1 = (plot function)
    - Press 2nd then TRACE
    - Select 2: zero
    - Move cursor near x-intercept, select left boundary and right boundary then press enter again.

4.9 Transformations

- Rewriting a Radical Function
  - If you simplify the radical so that x has a coefficient of 1, you can graph the function using its transformations.
  - $y = a\sqrt[n]{bx + c} + k$
    - Solve so b = 1
    - Then c = h