UNIT 5
SIMILARITY AND CONGRUENCE

M2 Ch. 2, 3, 4, 6 and M1 Ch. 13

5.1 Parallel Lines

- Objective
  - When parallel lines are cut by a transversal, I will be able to identify angle relationships, determine whether angles are congruent, supplementary, or both, and combine the theorems/postulates with algebra to solve for angle measures.
- Vocabulary

o Same-Side Interior Angles Postulate	o Alternate Interior Angles Postulate
o Alternate Exterior Angles Postulate	o Corresponding Angles Postulate

5.1 Parallel Lines - Extras

- Same-side interior angles: angles on the same side of the transversal inside the parallel lines
- Alternate Interior Angles: Angles on opposite sides of the transversal, inside the parallel lines
- Corresponding Angles: Angles on the same side of the transversal, on different intersections, one inside, one outside the parallel lines
- Alternate Exterior Angles: Angles on opposite sides of the transversal and are outside the parallel lines
- Vertical Angles: Angles that share a vertex and are opposite

- Vertical Angles Theorem: Vertical angles are congruent

5.1 Parallel Lines

- Identifying Angle Relationships
  - The special angle pairs formed by parallel lines and a transversal are congruent, supplementary, or both.
    - Supplementary (sum of two angles = 180°):
      - Same-Side Interior Angles Postulate
      - If a transversal intersects two parallel lines, then same-side interior angles are supplementary.
      - $m\angle 4 + m\angle 5 = 180$ and $m\angle 3 + m\angle 6 = 180$

5.1 Parallel Lines

- Congruent (angles have the same measure):
  - Alternate Interior Angles Theorem
    - If a transversal intersects two parallel lines, then alternate interior angles are congruent.
    - $\angle 4 \cong \angle 6$ and $\angle 3 \cong \angle 5$
  - Corresponding Angles Theorem
    - If a transversal intersects two parallel lines, then corresponding angles are congruent.
    - $\angle 1 \cong \angle 5$, $\angle 4 \cong \angle 8$, $\angle 2 \cong \angle 6$, and $\angle 3 \cong \angle 7$

5.1 Parallel Lines

  - Alternate Exterior Angles Theorem
    - If a transversal intersects two parallel lines, then alternate exterior angles are congruent.
    - $\angle 1 \cong \angle 7$ and $\angle 2 \cong \angle 8$
- Finding Measures of Angles
  - You can combine theorems and postulates with your knowledge of algebra to find angle measures.

5.2 Congruent Triangles I

- Objective
  - *I will be able to identify congruent figures and corresponding parts of congruent figures. I will be able to determine side and angle measure based on congruent figures. I will be able to prove two triangles congruent using SSS, SAS, and congruence transformations*.
- Vocabulary

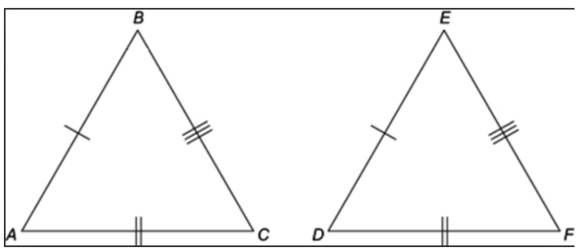
○ Congruent	○ SSS	○ SAS	○ Third Angle Theorem
○ Congruence Transformations		○ Congruent Polygons	

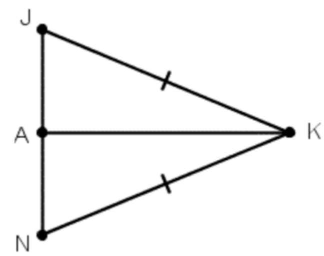
5.2 Congruent Triangles I

- Congruent figures have the same size and shape.
  - Can do compositions of rigid motions to one figure to map it onto the other.
  - Congruent polygons have congruent corresponding parts – matching sides and angles.
  - When naming congruent polygons, the corresponding vertices must be listed in the SAME ORDER.
- Third Angles Theorem
  - If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.
  - Remember: Angles of any triangle add up to 180°.

5.2 Congruent Triangles I

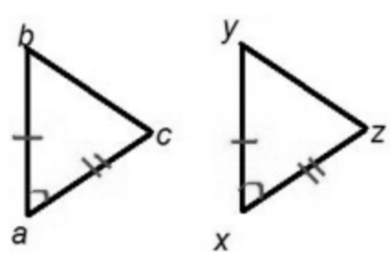
- Triangle Congruence Shortcuts
  - Side-Side-Side (SSS) Postulate
    - If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.
    - Indicates rigidity of triangles – architects and engineers rely on this!





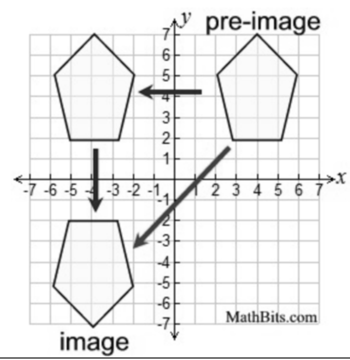
5.2 Congruent Triangles I

- Side-Angle-Side (SAS) Postulate
  - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.
  - Included angle refers to the angle formed by the two sides as its rays.



5.2 Congruent Triangles I

- Congruence Transformations
  - Two figures are congruent if and only if there is a sequence of one or more rigid motions that maps one figure onto the other.
  - Compositions of rigid motions that prove congruency are called congruence transformations.



5.3 Congruent Triangles II

- Objective

– I will be able to prove two triangles congruent using ASA, AAS, and/or HL. I will be able to identify the hypotenuse and legs of a right triangle. I will be able to recognize and use the fact that corresponding parts of congruent triangles are congruent.

- Vocabulary

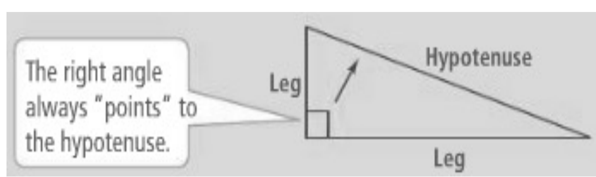
○ ASA	○ AAS	○ CPCTC	○ HL
○ Hypotenuse	○ Legs of a Right Triangle		○

5.3 Congruent Triangles II

- Triangle Congruence Shortcuts
  - Angle-Side-Angle (ASA) Postulate
    - If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.
    - An included side is the shared side of the two angles (between the angles).
  - Angle-Angle-Side (AAS) Theorem
    - If two angles and an nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

5.3 Congruent Triangles II

- Hypotenuse-Leg (HL) Theorem
  - *Anatomy of a right triangle*



  - *Conditions:*
    - Two right triangles
    - Triangles have congruent hypotenuses
    - One pair of congruent legs
- CPCTC - Corresponding Parts of Congruent Triangles are Congruent

5.3 Congruent Triangles II - HONORS

- Writing Proofs
  - Three types of proofs:
    - Two-Column
    - Flow
    - Paragraph
  - Proofs use theorems and postulates to support the statements that will get you from the given statement to the prove statement.
  - Use provided diagrams to identify any theorems/postulates that might be used.
  - Then, using the theorems/postulates, make statements that provide a logical path from a given to the statement being proved.

5.4 Similar Figures

- Objective
- I will be able to identify similar figures and use the scale factor to find the original sizes. I will be able to dilate figures and use dilations and scale factors to work out real-world problems.

- Vocabulary

○ Similar Figures	○ Similar Polygons	○ Extended Proportions	
○ Scale Drawing	○ Scale	○ Dilation	○ Center of Dilation
○ Enlargement	○ Reduction	○ Scale Factor	

5.4 Similar Figures

- Similarity
  - Similar figures have the same shape but not necessarily the same size.
  - Symbol: ~
  - Two polygons are similar polygons if corresponding angles are congruent and if lengths of corresponding sides are proportional.
  - Extended proportion: three or more equal proportions.

5.4 Similar Figures

- Scale
  - Scale factor, n: the ratio of corresponding linear measurements of two similar figures.
  - In a scale drawing, all lengths are proportional to their corresponding actual lengths.
    - Scale is the ratio that compares each length in a scale drawing to the actual length.
    - Scale can use different units (ex: 1cm = 50km)

5.4 Similar Figures

- Dilations (Review)
  - *Produce similar figures.*
  - *Two types:*
    - Enlargement: makes a larger figure (n > 1)
    - Reduction: makes a smaller figure (0 < n < 1)
  - *Dilations and scale factors can help you understand real-world enlargements and reductions.*

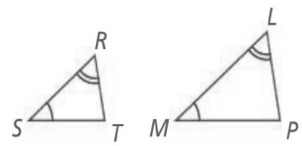
5.5 Triangle Similarity

- Objective
- I will be able to use postulates and theorems to identify similar triangles. I will be able to use indirect measurement to find actual lengths.

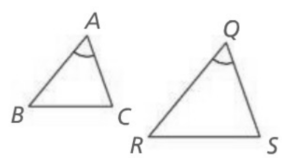
- Vocabulary

○ Indirect Measure	○ Angle-Angle Similarity Postulate
○ Side-Angle-Side Similarity Postulate	○ Side-Side-Side Similarity Postulate

5.5 Triangle Similarity



- Similarity Theorems/Postulates
  - *Angle-Angle Similarity (AA~) Postulate*
    - If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
  - *Side-Angle-Side Similarity (SAS~) Theorem*
    - If an angle of one triangle is congruent to an angle of a second triangle, and the sides that include the two angles are proportional, then the triangles are similar.



5.5 Triangle Similarity

- *Side-Side-Side Similarity (SSS~) Theorem*
  - If the corresponding sides of two triangles are proportional, then the triangles are similar.
- Finding Lengths
  - *Similar triangles can be used to find lengths that cannot be measured easily.*
  - *Indirect measurement – a method of measurement that uses formulas, similar figures and/or proportions.*

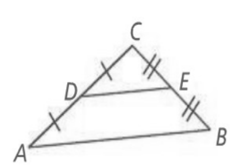
5.6 Midsegment & Side-Splitter Theorems

- Objective
  - I will be able to identify a midsegment, use the triangle midsegment theorem, the side-splitter theorem and its corollary, and the triangle-angle-bisector theorem to solve for missing sides and variables.
- Vocabulary

○ Midsegment	○ Triangle Midsegment Theorem	○ Side-Splitter Theorem
○ Triangle-Angle-Bisector Theorem	○ Corollary to Side-Splitter Theorem	

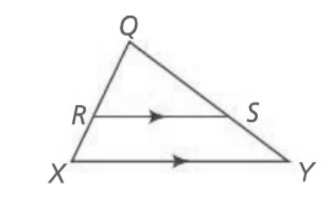
5.6 Midsegment & Side-Splitter Theorems

- Midsegment
  - *A midsegment of a triangle is a segment connecting the midpoints of two sides of a triangle.*
  - *Triangle Midsegment Theorem*
    - If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long.
    - Can be used to find the lengths of segments that might be difficult to measure directly.



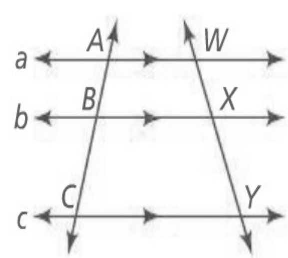
5.6 Midsegment & Side-Splitter Theorems

- Side-Splitter
  - *Side-Splitter Theorem*
    - If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.
    - For the diagram below, *XR/RQ=YS/SQ*



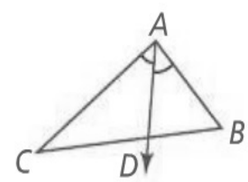
5.6 Midsegment & Side-Splitter Theorems

- *Corollary to the Side-Splitter Theorem*
  - If three parallel lines intersect two transversals, then the segments intercepted on transversals are proportional.
  - In the diagram below, *AB/BC=WX/XY*



5.6 Midsegment & Side-Splitter Theorems

- *Triangle-Angle-Bisector Theorem*
  - If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.
  - In the diagram to the left, *CD/DB=CA/BA*



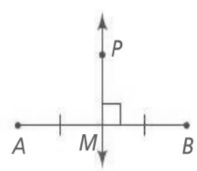
5.7 Bisectors

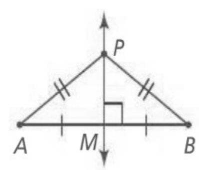
- Objective
- I will be able to recognize a perpendicular bisector and an angle bisector. I will be able to use the associated theorems to find missing angles, sides, and variables.
- Vocabulary

○ Equidistant	○ Bisector	○ Perpendicular Bisector Theorem
○ Angle Bisector Theorem ○ Distance from a point to a line		○ Converse of Perpendicular Bisector Theorem

5.7 Bisectors

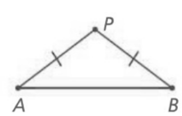
- Using the Perpendicular Bisector Theorem
  - *There is a special relationship between the points on the perpendicular bisector of a segment and the endpoints of the segment.*
  - *Equidistant – the same distance*
  - *Perpendicular Bisector Theorem*
    - If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

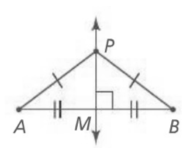




5.7 Bisectors

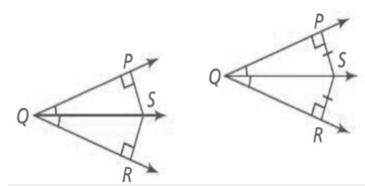
  - *Converse of the Perpendicular Bisector Theorem*
    - If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.





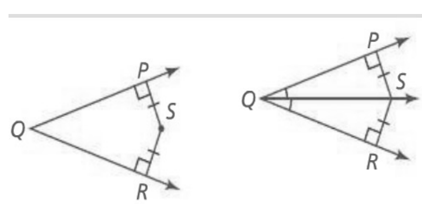
5.7 Bisectors

- Using the Angle Bisector Theorem
  - *The distance from a point to a line is the length of the perpendicular distance from the point to the line.*
  - *Shortest length from the line to the point.*
  - *Angle Bisector Theorem*
    - If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.



5.7 Bisectors

- *Converse of the Angle Bisector Theorem*
  - If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector.



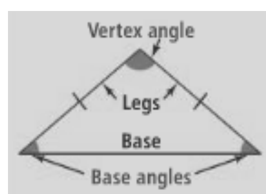
5.8 Isosceles Triangles

- Objective
  - *I will be able to identify parts of an isosceles triangle. I will be able to use the theorems and corollaries associated with isosceles triangles to find missing sides, missing angles, and variables.*
- Vocabulary

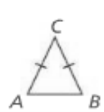
<table>
<tr><td colspan="2">○ Legs of an isosceles triangle</td><td>○ Base of an isosceles triangle</td></tr>
<tr><td colspan="2">○ Vertex angle of an isosceles triangle</td><td>○ Base angle of an isosceles triangle</td></tr>
<tr><td colspan="2">○ Equilateral Triangle</td><td>○ Equiangular triangle</td></tr>
<tr><td colspan="2">○ Converse of Isosceles Triangle Theorem</td><td>○ Isosceles Triangle Theorem</td></tr>
<tr><td colspan="2">○ Corollary to Isosceles Triangle Theorem</td><td rowspan="2">○ Corollary to Converse of Isosceles Triangle Theorem</td></tr>
<tr><td>○ Corollary</td><td>○ Theorem 22</td></tr>
</table>

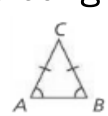
5.4 Isosceles Triangles

- Anatomy of an Isosceles Triangle



- Theorems
  - *Isosceles Triangle Theorem*
    - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
  - *Converse of the Isosceles Triangle Theorem*
    - If two angles of a triangle are congruent, then the sides opposite those angles are congruent.





5.4 Isosceles Triangles

- *Theorem 22*
  - If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.

- Corollaries
  - *A corollary is a theorem that can be proved easily using another theorem.*
  - *Can be used as a reason in a proof*

5.4 Isosceles Triangles

- *Corollary to the Isosceles Triangle Theorem*
  - If a triangle is equilateral, then the triangle is equiangular.
  - Equilateral – all sides are congruent to each other
  - Equiangular – all angles are congruent to each other
- *Corollary to the Converse of the Isosceles Triangle Theorem*
  - If a triangle is equiangular, then the triangle is equilateral.