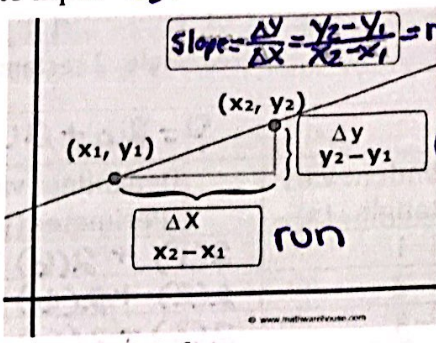
Unit 2 – Linear Functions

2.1. Patterns and Linear Functions

- Variables
  - Independent → input (x)
  - Dependent → output → changes due to input (y)
- Equation of a Line (slope-intercept form)

$y = mx + b$

dependent, slope, independent, y-intercept



rise

$m = \frac{rise}{run}$

  - To find the x-intercept, substitute 0 for y and solve for x.
  - ex: Find the slope, y-intercept, and x-intercept of the line $y = 3x + 4$.

$\boxed{m = 3 \quad b = 4}$ $\quad 0 = 3x + 4 \quad -4 = 3x \quad \boxed{\frac{-4}{3} = x}$

  - ex: Find the slope of the line that intercepts (0,3) and (5,4).

$m = \frac{4-3}{5-0} = \boxed{\frac{1}{5}}$

  - ex: Find the slope, y-intercept, and x-intercept of the line $10x + 5y = 15$.

$5y = 15 - 10x$

$y = 3 - 2x$

$\boxed{m = -2 \quad b = 3}$

$10x + 5(0) = 15$

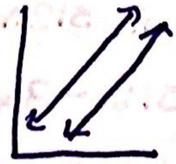
$10x = 15$

$\boxed{x = 1.5}$

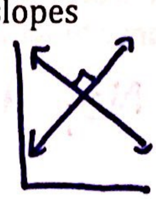
- Horizontal and Vertical Lines
  - Vertical lines have the equation x = #.
    - No y-intercept
    - Undefined slope

} Not Functions!

  - Horizontal lines have the equation y = #
    - Y-intercept = #
    - No x-intercept
    - Slope = 0
- Parallel and Perpendicular Lines
  - Parallel lines have the <u>same</u> slope
    - do not intersect
    - Ex: $y = 2x + 3$ and $y = 2x + 6$



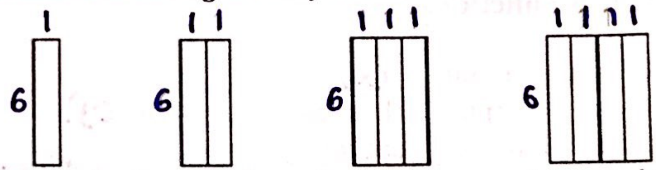
  - Perpendicular lines have negative reciprocal slopes
    - intersect and right angles
    - ex: $y = 2x + 3$ and $y = -\frac{1}{2}x + 4$



neg reciprocal → <u>flip</u>

& multiply by −1

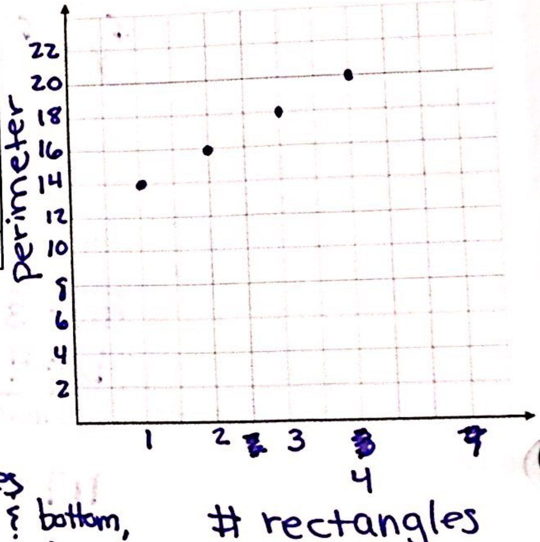
- Using patterns to find and plot linear functions
  - ex: What is the relationship between the number of rectangles and the perimeter of the figure they form?



1 rectangle 2 rectangles 3 rectangles 4 rectangles

$P = 2w + 2l$

Independent var. # rectangles (x)	Dependent var. Perimeter (y)	Ordered Pairs
1	$2(1) + 2(6) = 14$	(1,14)
2	$2(2) + 2(6) = 16$	(2,16)
3	$2(3) + 2(6) = 18$	(3,18)
4	$2(4) + 2(6) = 20$	(4,20)

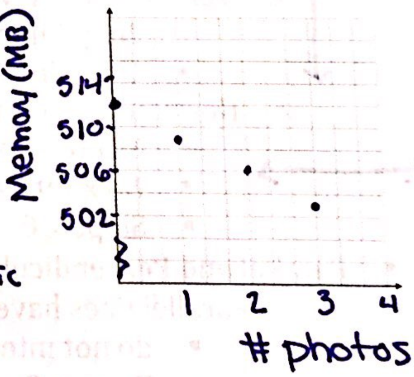


Equation: $y = 2x + 12$

Statement: Multiply the # rectangles by 2 to get the total length of top & bottom, then add 12 to account for the left & right sides.

- Functions
  - Relationships that pair each input value with EXACTLY ONE output value.
  - Linear Functions: functions whose graph is a non-vertical line or part of a non-vertical line.
  - Ex: Determine the equation of the function below, give a statement describing it, and graph it.

# photos (x)	Memory (MB) (y)	Ordered Pairs
+1 0	512 -3	(0,512)
+1 1	509 -3	(1,509)
+1 2	506 -3	(2,506)
3	503	(3,503)



amount of memory is 512MB minus 3MB/pic

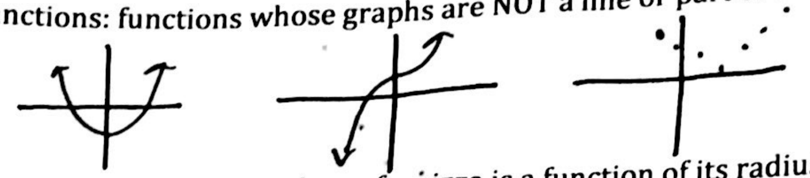
$y = 512 - 3x$

  - Ex: Does the set of ordered pairs (0,2), (1,4), (3,5), and (1,8) represent a linear function? Explain.

No; there is more than one output for $x = 1$

2.2. Patterns and Nonlinear Functions

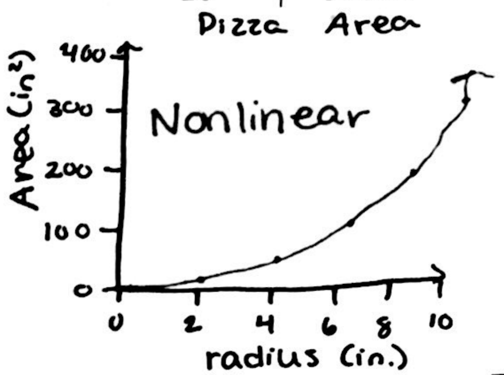
- Nonlinear Functions: functions whose graphs are NOT a line or part of a line.

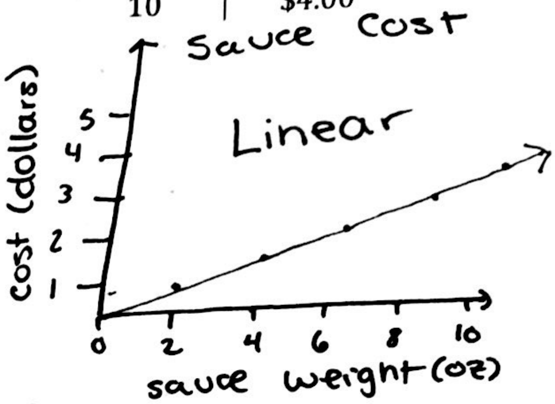


  - Ex: The area, A, in square inches, of a pizza is a function of its radius, r, in inches. The cost, C, in dollars, of the sauce for a pizza is a function of the weight, w, in ounces, of the sauce used. Graph these functions shown by the tables. Is each one linear or nonlinear?

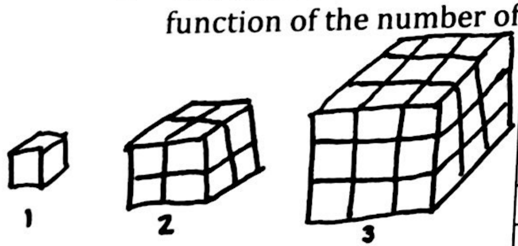
r (in)	A (in^2)
2	12.57
4	50.27
6	113.10
8	201.06
10	314.16

w (oz)	C (dollars)
2	$0.80
4	$1.60
6	$2.40
8	$3.20
10	$4.00





- Representing Patterns & Nonlinear Functions.
  - The table shows the total number of blocks in each figure below as a function of the number of blocks in one edge.



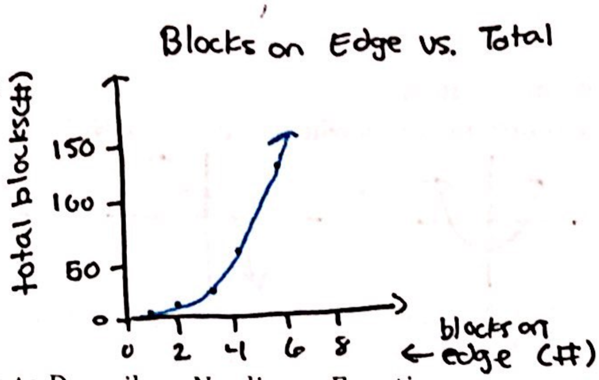
# blocks (edge)	Total # blocks	Ordered pair
1	1	(1,1)
2	8	(2,8)
3	27	(3,27)
4	64	(4,64)
5	125	(5,125)

Represent the relationship you can use to complete the table with words, an equation, and a graph.

Words: The total # of blocks, y, is the cube of the # blocks on one edge, x.

Equation: $y = x^3$

Graph:



- Writing a Rule to Describe a Nonlinear Function
  - A function can be thought of as a rule that you apply to the input to get the output.
  - Ex: The ordered pairs (1.2). (2,4), (3, 8), (4,16), (5,32) represent a function. What is the rule that represents this function?

x	y
1	2
2	4
3	8
4	16
5	32

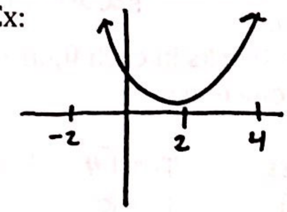
→ possible rules for (1,2): $y=2x$, $y=x+1$, $y=2^x$

→ $x+1$ does not work for these. $y=2x$ works for (2,4), but not (3,8). $y=2^x$ works for both.

→ can rewrite all y coordinates as $2^1, 2^2, 2^3, 2^4, 2^5$ so the pattern matches.

Rule: $\boxed{y=2^x}$

- Intervals
  - Use closed intervals [or] if a point is included in the interval.
  - Use open intervals (or) if a point is not included in the interval.
  - These can be mixed & matched as needed.
  - Ex:

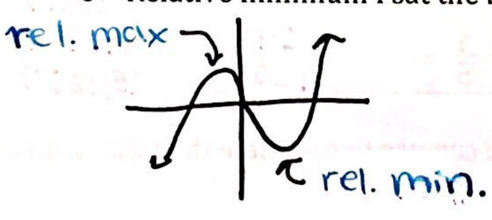


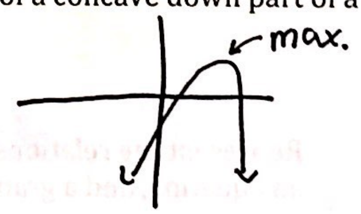
a. open intervals of increase: $(2, \infty)$

b. open intervals of decrease: $(-\infty, 2)$

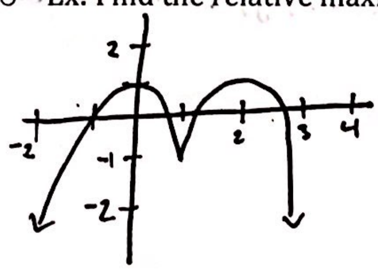
c. open intervals where constant: None

- Maxima & Minima
  - Relative maximum is at the top of a concave down part of a curve
  - Relative minimum I sat the bottom of a concave down part of a curve.





  - Ex: Find the relative maxima/minima, if any.



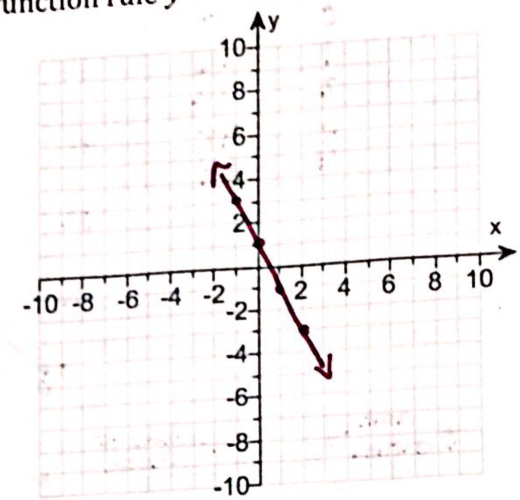
a. maxima: (0,1) & (2,1)

b. minimum: (1, -1)

2.3. Graphing Functions

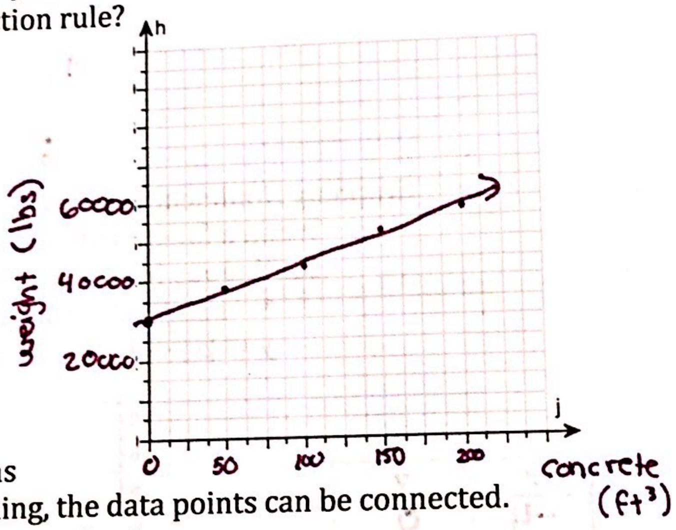
- Graphing a Function Rule
  - The set of all solutions to a function forms an equation's graph.
  - A real-world graph should only show points that make sense in the given situation.
    - Ex: What is the graph of the function rule $y = -2x + 1$

x	y = -2x + 1	Ordered pairs
-1	-2(-1)+1 = 3	(-1, 3)
0	-2(0)+1 = 1	(0, 1)
1	-2(1)+1 = -1	(1, -1)
2	-2(2)+1 = -3	(2, -3)

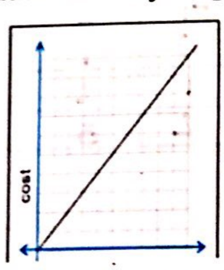


- Graphing a Real-World Function Rule
  - Choose appropriate intervals for the units on the axes.
  - If all data is nonnegative, show only 1st quadrant.
    - Ex: The function rule $W = 146c + 30000$ represents the total weight W, in pounds, of a concrete mixer truck that carries c cubic feet of concrete. If the capacity of the truck is about 200ft? What is a reasonable graph of the function rule?

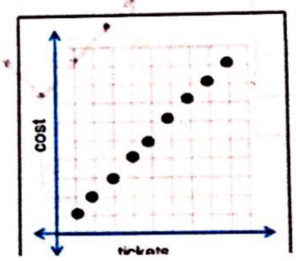
x	W = 146c + 30000	Ordered pairs
0	146(0)+30000 = 30000	(0, 30000)
50	146(50)+30000 = 37300	(50, 37300)
100	146(100)+30000 = 44600	(100, 44600)
150	146(150)+30000 = 51900	(150, 51900)
200	146(200)+30000 = 59200	(200, 59200)



- Identifying Continuous and Discrete Graphs
  - If any point between data has meaning, the data points can be connected.
  - Some data can only be graphed as isolated points.



continuous graph
(unbroken)

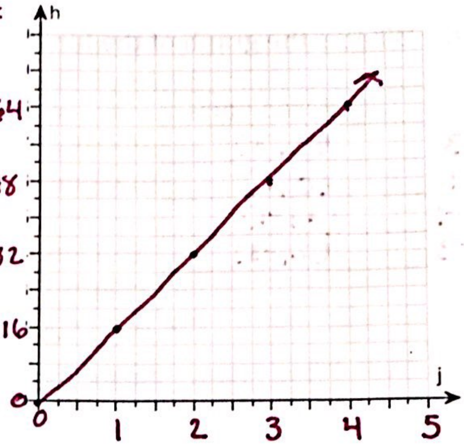


discrete graph
(isolated pts)

- Ex: A local cheese maker is making cheddar cheese to sell at a farmer's market. The amount of milk used to make the cheese and the price at which he sells the cheese are shown below. Graph each function. Is the graph continuous or discrete? Explain.

weight of cheese $w = 16m$ gal. milk

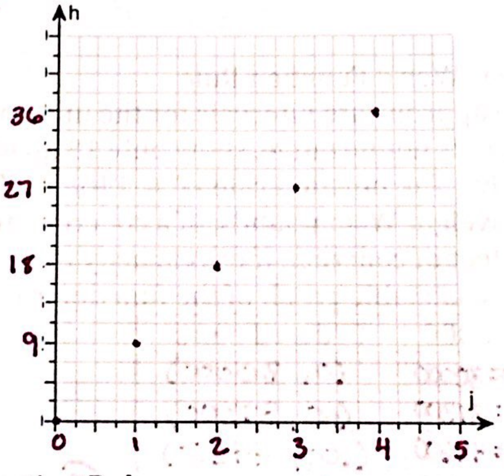
m	w
0	0
1	16
2	32
3	48
4	64



any amount of milk makes sense, so it's continuous.

amt. \$ earned $a = 9n$ wheels of cheese

n	a
0	0
1	9
2	18
3	27
4	36

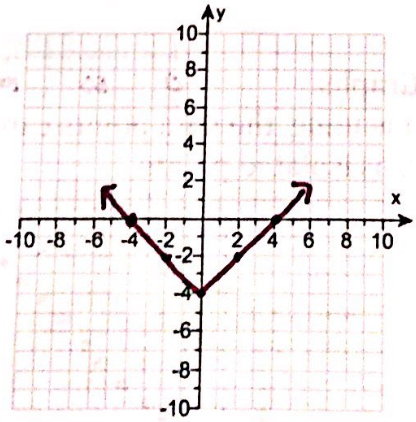


can only sell whole wheels of cheese, so it's discrete.

- Graphing Nonlinear Function Rules
  - When a function does not represent a real-world situation, graph it as a continuous function.
    - Ex: What is the graph of each function rule?

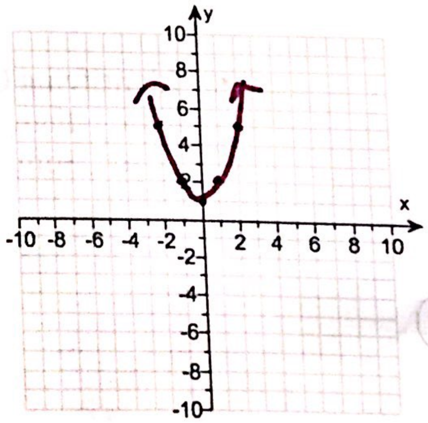
$y = |x| - 4$

x	y
−4	0
−2	−2
0	−4
2	−2
4	0



$y = x^2 + 1$

x	y
−2	5
−1	2
0	1
1	2
2	5



2.4. Writing a Function Rule (M1 2.5)

- Writing a Function Rule
  - Many real-world functional relationships can be represented by equations, which can be used to find solutions.
    - Ex: You can estimate the temperature by counting the number of chirps of the snowy tree cricket. The outdoor temperature is about 40°F more than ¼ the number of chirps the cricket makes in one minute. What is a function rule that represents this situation?

T = temperature
n = # chips

Temp. is 40 more than one quarter number of chirps

$T = 40 + \frac{1}{4}n$

$\boxed{T = 40 + \frac{1}{4}n}$

    - Ex: A worker's earnings, e, are a function of the number of hours, n, worked at a rate of \$8.75 per hour. Write a function that represents this situation.

earnings are fn of # hrs worked @ rate of 8.75

$e = n \cdot \$8.75$

$\boxed{e = 8.75n}$

- Writing and Evaluating a Function Rule
  - A kennel charges \$15 per day to board a dog. Upon arrival, each dog must have a flea bath that costs \$12. Write a function rule for the total cost for n days of boarding plus a bath. How much does a 10-day stay cost?

d = # days
c = total cost

cost is \$15/day and flea bath

$c = \$15d + 12$

10-day cost: $c = 15(10) + 12 = \boxed{\$162}$

- Writing a Nonlinear Function Rule
  - Write a function rule for the area of a rectangle whose length is 5ft more than its width. What is the area of the rectangle w ?

l

w A

$A = l \cdot w$

$\downarrow$

$l = w + 5$

$\downarrow$

w + 5

w A

$A = w(w+5) = w^2 + 5w$

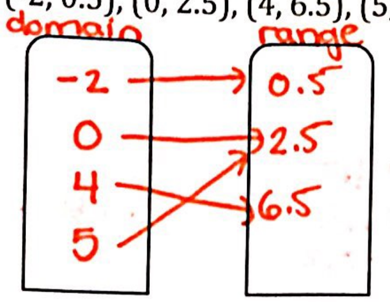
$w = 9$

$A = (9)^2 + 5(9)$

$\boxed{A = 126\text{ ft}^2}$

2.5. Determining if an Equation is a Function (M1 2.6)

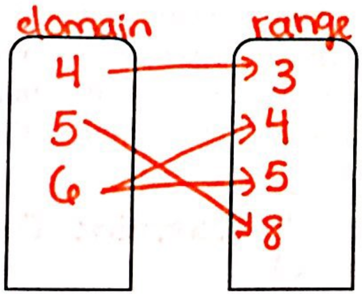
- A relation is a pairing of numbers in one set, called the domain, with numbers in another set, called the range.
  - Domain → set of x-values
  - Range → set of y-values
  - Often represented as ordered pairs (x,y)
  - Ex: Identify the domain and range of each relation. Represent the relation with a mapping diagram. Is the relation a function?
    - (-2, 0.5), (0, 2.5), (4, 6.5), (5, 2.5)



each domain value is paired to only 1 range value.

The relation is a function.

    - (6, 5), (4, 3), (6, 4), (5, 8)

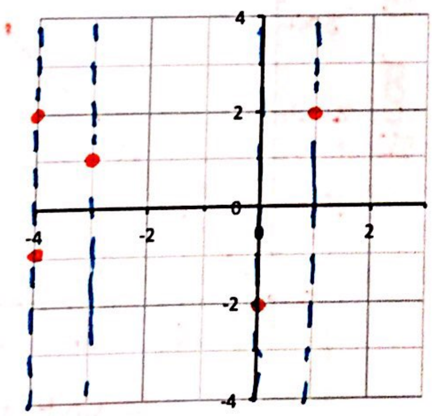


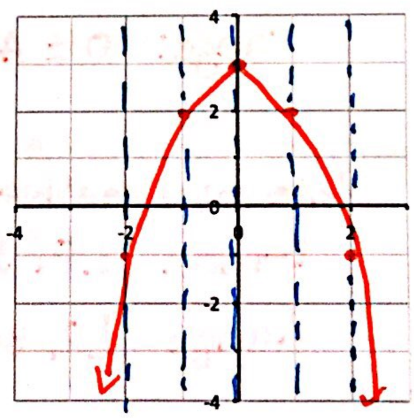
Domain value 6 is mapped to 2 range values.

The relation is NOT a fn.

- Identifying Functions Using the Vertical Line Test
  - Vertical Line Test: If any vertical line passes through more than one point on the graph, then for some domain value there is more than one range value. The relation is not a function.
  - Ex: Is the relation a function? Use the Vertical Line Test.

(-4, 2), (-3, 1), (0, -2), (-4, -1), (1, 2) $\quad\quad y = -x^2 + 3$





x	y
-1	2
-2	-1
0	3
1	2
2	-1

Not a function

domain value 6 corresponds to two range values 2 & -1.

Function

No vertical line passes through more than 1 pt on the graph.

- Evaluating a Function
  - In function notation, f(x) replaces y
    - $y = -3x + 1$ becomes $f(x) = -3x + 1$
    - f(x) is read as "f of x"
    - Other letters can be used, such as g(x) or h(x)
    - Ex: The function $w(x) = 250x$ represents the number of words w(x) you can read in x minutes. How many words can you read in 8 minutes? $w(8) = 250(8) = \boxed{2000 \text{ words}}$
- Finding the Range of the Function
  - Use the x-values provided to find the corresponding y-values.
  - Ex: The domain of $f(x) = -1.4x + 4$ is {1, 2, 3, 4}. What is the range?

x	f(x)
1	$-1.4(1)+4 = 2.6$
2	$-1.4(2)+4 = 1.2$
3	$-1.4(3)+4 = -0.2$
4	$-1.4(4)+4 = -1.6$

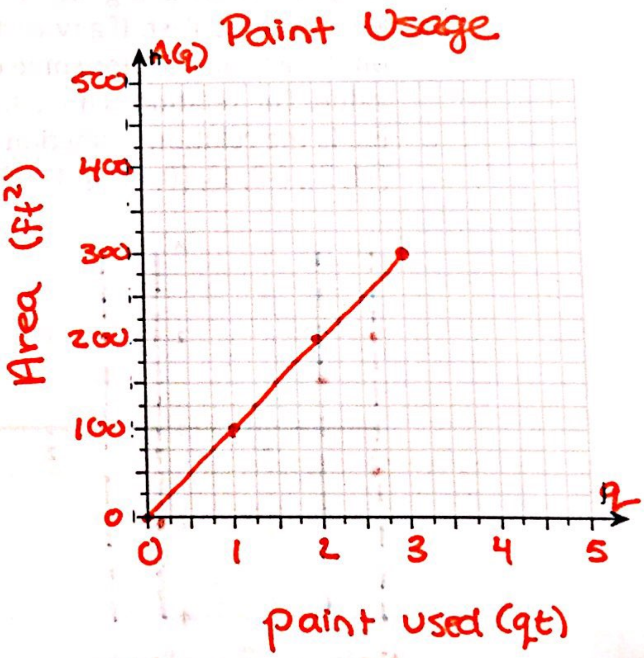
$\boxed{\text{Range: } \{-1.6, -0.2, 1.2, 2.5\}}$

- Identifying Reasonable Domain and Range
  - Ex: You have 3qt of paint to paint the trim in your house. A quart of paint covers 100ft². The function $A(q) = 100q$ represents the area A(q), in square feet, that q quarts of paint cover. What domain and range are reasonable for the function? What is the graph of the function?

Least amount: 0qt
Most amount: 3qt
} domain: $0 \leq q \leq 3$

q	A(q)
0	$100(0)=0$
1	$100(1) = 100$
2	$100(2) = 200$
3	$100(3) = 300$

range: $0 \leq A(q) \leq 300$



Interval Notation:
Domain $[0, 3]$
Range $[0, 300]$

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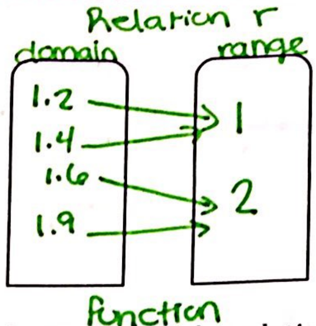
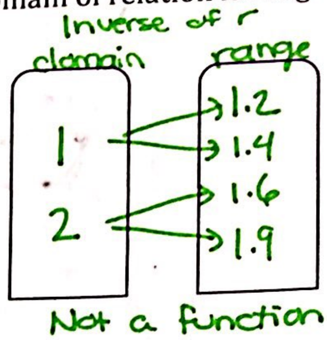
2.6. Inverse Linear Functions (M1 6.7)

- If a relation pairs element a of its domain to element b of its range, the inverse relation pairs b with a.
- If (a,b) is an ordered pair of the relation, then (b,a) is an ordered pair of its inverse.
    - The diagram shows a relation r (a function) and its inverse (not a function).
    - Range of relation is domain of function, and domain of relation is range of function.





- Ex: What is the inverse of the relation?

Relation s

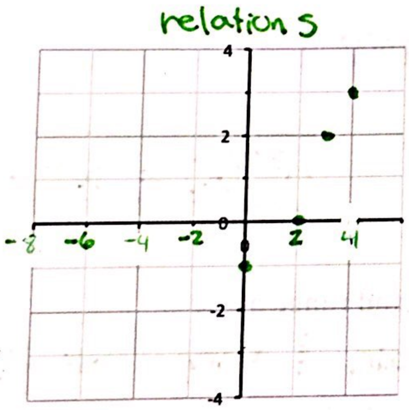
x	y
0	-1
2	0
3	2
4	3

switch x & y values →

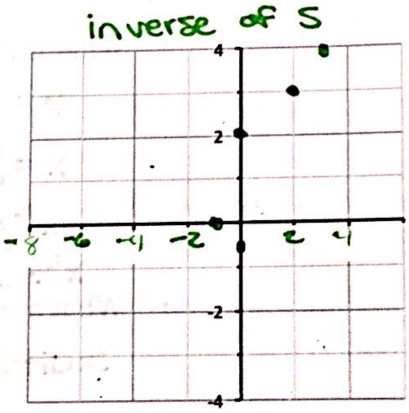
Inverse of Relation s

x	y
-1	0
0	2
2	3
3	4

- What are the graphs of s and its inverse?



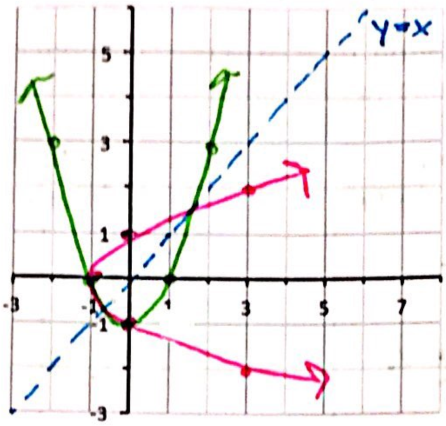
reflection over y=x →



- Finding an Equation for the Inverse
    - The graphs of the relation and its inverse are reflections of each other over y = x. If you describe a relation or function by an equation in x and y, you can switch x and y to get an equation for the inverse.
    - Ex: What is the inverse of the relation described by $y = x^2 - 1$?

$$x = y^2 - 1$$
$$+1 \qquad +1$$
$$\sqrt{x+1 = y^2}$$
$$\boxed{\pm\sqrt{x+1} = y}$$

- Graphing a Relation and its Inverse
  - Ex: What are the graphs of $y = x^2 - 1$ and its inverse, $y = \pm\sqrt{x+1}$?



x	$y=x^2-1$
-2	3
-1	0
0	-1
1	0
2	3

x	$y=\pm\sqrt{x+1}$
3	-2
-1	0
0	-1
0	1
3	2

- Finding an Inverse Function
  - The inverse of a function f is denoted by f^{-1}.
  - You read f^{-1} as "the inverse of f"
  - f^{-1} may or may not be a function, even if f(x) is a function.
    - Ex: Consider the function $f(x) = \sqrt{x-2}$
      - What are the domain and range of f?

        The radicand cannot be negative, so $\boxed{x \geq 2}$

        The principal sq. root is nonnegative, so $\boxed{y \geq 0}$

      - What is f^{-1}, the inverse of f?

        $y = \sqrt{x-2}$

        $x = \sqrt{y-2}$

        $x^2 = y-2$

        $x^2+2 = y$

        $\boxed{f^{-1}(x) = x^2+2, \text{ for } x \geq 0}$

      - What are the domain and range of f^{-1}?

        Switch domain & range of f.

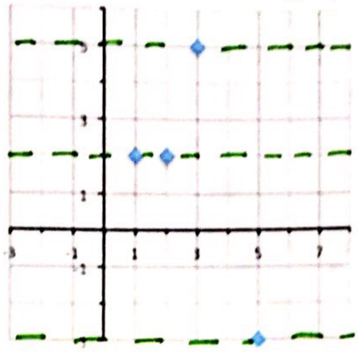
        domain: $\boxed{x \geq 0}$

        range: $\boxed{y \geq 2}$

      - Is f^{-1} a function? Explain.

        For each x in the domain ($x \geq 0$) of f^{-1}, there is only one value of y in the range. Therefore, $f^{-1}(x) = x^2+2$ for $x \geq 0$ is a function.

- Horizontal line test – If any horizontal line passes through more than one point on the graph of f(x), then for some domain value there is more than one range value for f^{-1}. The inverse of the relation is not a function.
  - Ex: Given the graph of f(x) below, is its inverse a function? Explain.



It's inverse is not a function; the horizontal line $y=2$ passes through more than one point.

- Finding an Inverse Formula
  - Functions that model real-world behavior are often expressed as formulas with meaningful variable, like $A = \pi r^2$ for the area of a circle.
  - Strictly speaking, the inverse would be $r = \pi A^2$, but this expresses a false relationship between A and r.
  - It is better to leave the variables in place and solve for r. $r = \sqrt{\frac{A}{\pi}}$ is inversely expressed.
  - Ex: The function $d = 4.9t^2$ represents the distance d, in meters, that an object falls in t seconds due to Earth's gravity. Find the inverse of this function. How long, in seconds, does it take for a cliff diver diving from 24 meters to reach the water below?

$d = 4.9t^2$ $\quad$ $d = 24m$

$\frac{d}{4.9} = t^2$ $\quad$ $\sqrt{\frac{24}{4.9}} = t = \boxed{2.2s}$

$\boxed{\sqrt{\frac{d}{4.9}} = t}$