Unit 6
Trigonometry

M2 2.5, 7.1-7.4

6.1 Interior and Exterior Angles

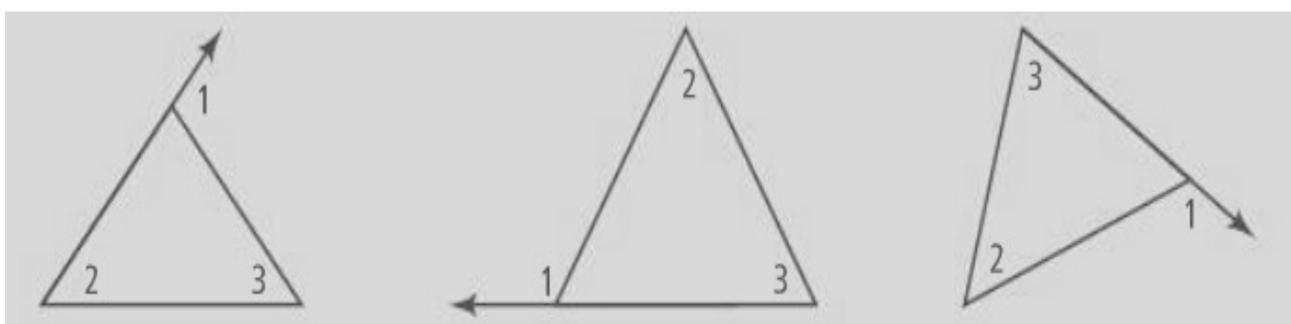
- *Obj.*: I will be able to identify both exterior angles and remote interior angles, and relate the measures of these two types of angles.

- **Vocabulary**

○ Exterior Angle of a Polygon	○ Remote Interior Angles

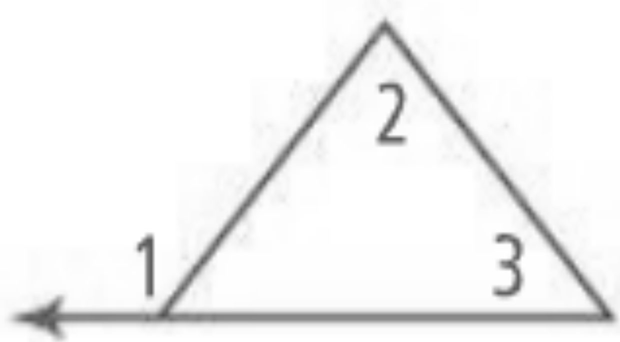
6.1 Interior and Exterior Angles

- Triangle-Angle-Sum Theorem (Review)
  - The sum of the three angles in any triangle is 180°.
- Exterior Angles
  - Exterior Angle of a Polygon – an angle formed by a side and an extension of an adjacent side.
  - Remote Interior Angles – the two nonadjacent interior angles



6.1 Interior and Exterior Angles

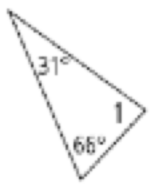
- Triangle Exterior Angle Theorem
  - The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.
  - In the diagram, $m\angle 1 = m\angle 2 + m\angle 3$



6.1 Interior and Exterior Angles - Practice

Find $m\angle 1$.

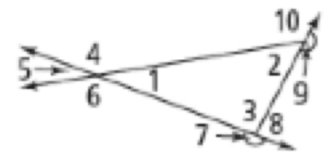
1.



5. a. Which of the numbered angles are exterior angles?

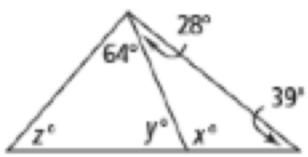
b. Name the remote interior angles for each exterior angle.

6. Which two exterior angles share the same remote interior angles? Explain.



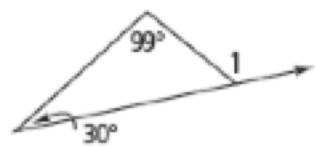
Algebra Find the value of each variable.

3.

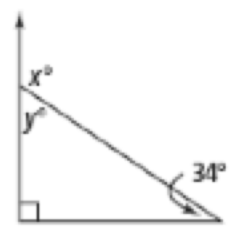


Algebra Find each missing angle measure.

7.

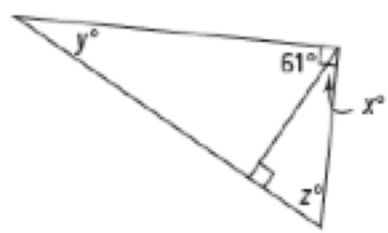


9. What are the values of x and y in the right triangle?



Find the values of the variables and the measures of the angles.

11.



12.

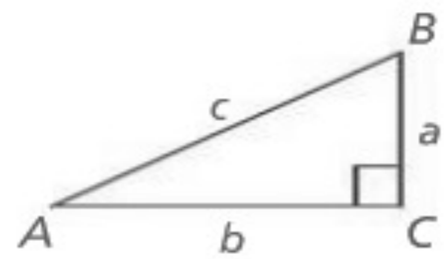
6.2 Pythagorean Theorem

- *Obj.*: I will be able to find the lengths of the legs and/or hypotenuse of a right triangle. I will be able to recognize Pythagorean triples, and I will be able to classify a triangle based on its sides.
- **Vocabulary**

○ Pythagorean Triple	○ Theorem 68	○ Theorem 67
○ Pythagorean Theorem	○ Converse of the Pythagorean Theorem	

6.2 Pythagorean Theorem

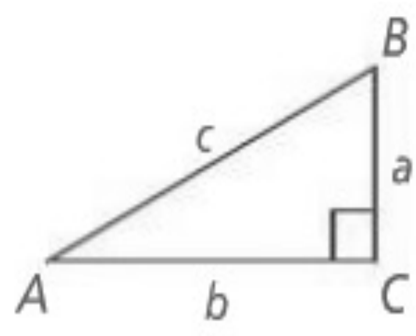
- Pythagorean Theorem
  - If you know any two sides of a right triangle, you can calculate the third.



  - Pythagorean Theorem
    - If a triangle is a right triangle, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse
    - $a^2+b^2=c^2$
    - Pythagorean Triple – a set of nonzero whole numbers a, b, and c that satisfy the Pythagorean theorem.

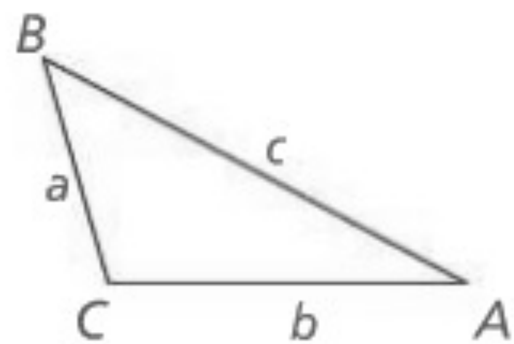
6.2 Pythagorean Theorem

- Classifying a Triangle
  - Right Triangle
    - Converse of the Pythagorean Theorem: If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.
    - $c^2 = a^2+b^2$



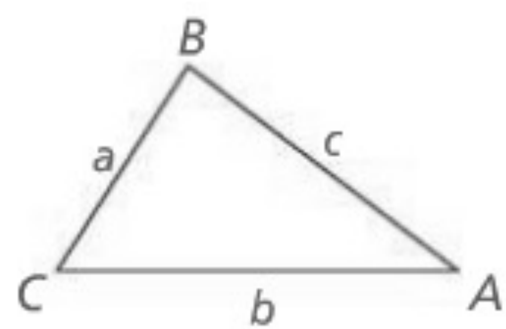
6.2 Pythagorean Theorem

- Obtuse Triangle
  - Theorem 67: If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse.
  - $c^2 > a^2+b^2$



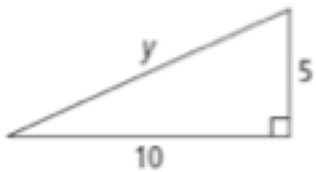
6.2 Pythagorean Theorem

- Acute Triangle
  - Theorem 68: If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute.
  - $c^2 < a^2+b^2$



6.2 Pythagorean Theorem - Practice

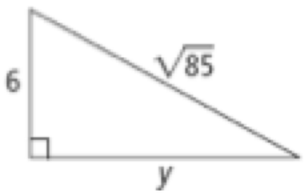
1.



5. 3, 8, 10

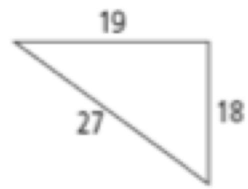
9. 13, 84

3.



Is each triangle a right triangle? Explain.

11.



7. A repairman leans the top of an 8-ft ladder against the top of a stone wall. The base of the ladder is 5.5 ft from the wall. About how tall is the wall? Round to the nearest tenth of a foot.

13. The playing surface of a football field is 300 ft long and 160 ft wide. If a player runs from one corner of the field to the opposite corner, how many feet does he run?

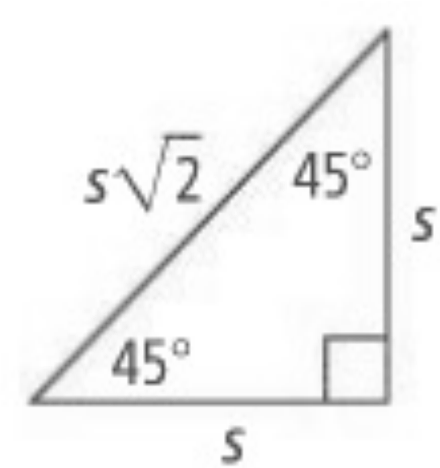
6.3 Special Right Triangles

- *Obj.*: I will be able to identify two types of special right triangles; I will be able to use the special right triangle theorems to solve for sides of special right triangles.
- **Vocabulary**

○ 45-45-90 Triangle Theorem	○ 30-60-90 Triangle Theorem

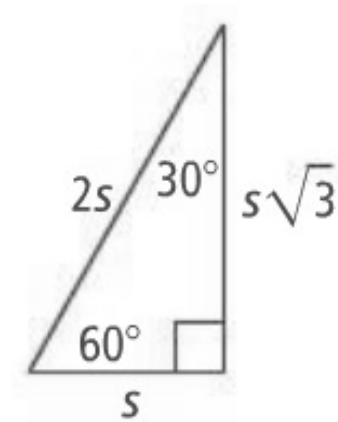
6.3 Special Right Triangles

- 45°–45°–90° Triangles
  - In a 45°–45°–90° triangle, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.
  - This is an isosceles triangle!
    - Both legs are congruent (*s*)
    - $H = s\sqrt{2}$



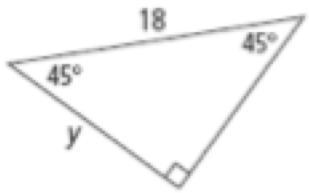
6.3 Special Right Triangles

- 30°–60°–90° Triangles
  - In a 30°–60°–90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.
    - $H = 2s$
    - $L = s\sqrt{3}$



6.3 Special Right Triangles - Practice

1.



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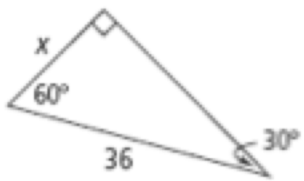

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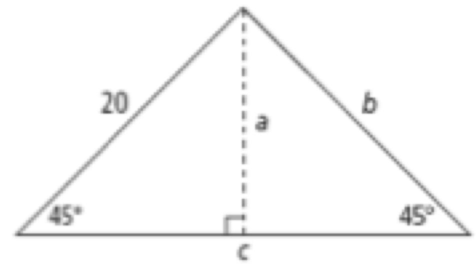

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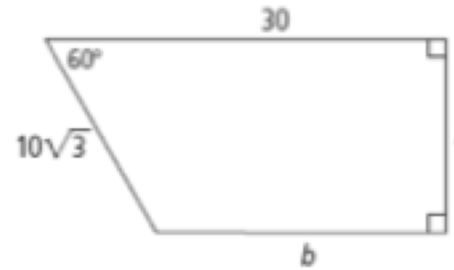


5. A square has side length 95. What is the length of the diagonal of the square? Express your answer in simplest radical form.

7. You set up a makeshift greenhouse by leaning a square pane of glass against a building. The glass is 4.5 ft long, and it makes a 30° angle with the ground. How much horizontal distance between the building and the glass is there to grow plants? Round to the nearest inch.

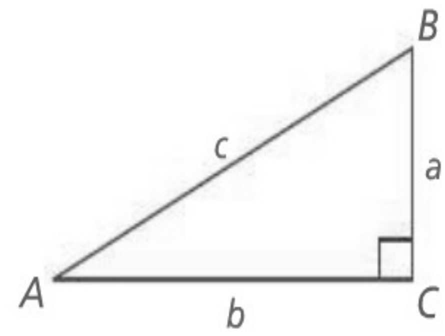
6.4 Primary Trig Ratios

- *Obj.:* I will be able to find side lengths and angles of triangles using trigonometric ratios and their inverses.

- **Vocabulary**

○ Trigonometric Ratios	○ Sine	○ Cosine	○ Tangent
○ Inverse Trigonometric Ratios	○ Arcsine	○ Arccosine	○ Arctangent

6.4 Primary Trig Ratios



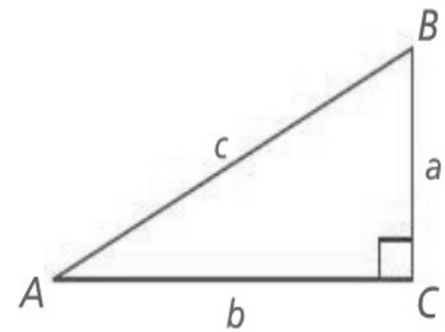
- Trigonometric Ratios
  - Used to find lengths of sides in a right triangle.
  - Similar right triangles have equivalent ratios for their corresponding sides called trigonometric ratios.

Sine: $\sin(\theta) = \dfrac{Opposite}{Hypotenuse}$ $\sin(A) = \dfrac{a}{c}$

Cosine: $\cos(\theta) = \dfrac{Adjacent}{Hypotenuse}$ $\cos(\mathrm{A}) = \dfrac{b}{c}$

Tangent: $\tan(\theta) = \dfrac{Opposite}{Adjacent}$ $\tan(\mathrm{A}) = \dfrac{a}{b}$

6.4 Primary Trig Ratios



- Inverse Trigonometric Ratios
  - Used to find angle measures in a right triangle.

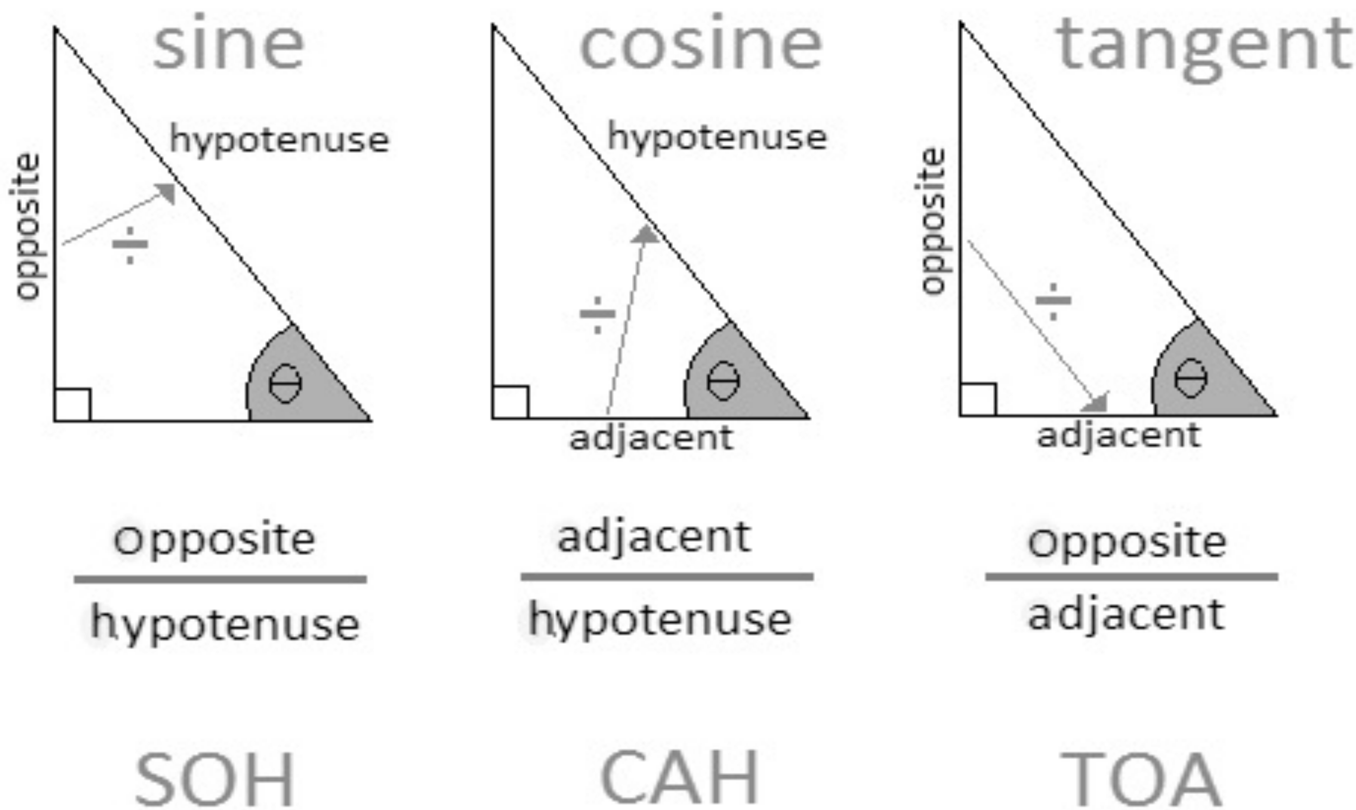
Arcsine: $\sin^{-1}\left(\frac{opposite}{hypotenuse}\right) = \theta$ $\sin^{-1}\left(\frac{a}{c}\right) = A$

Arccosine: $\cos^{-1}\left(\frac{adjacent}{hypotenuse}\right) = \theta$ $\cos^{-1}\left(\frac{b}{c}\right) = A$

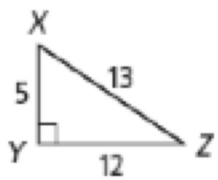
Arctangent: $\tan^{-1}\left(\frac{opposite}{adjacent}\right) = \theta$ $\tan^{-1}\left(\frac{a}{b}\right) = A$

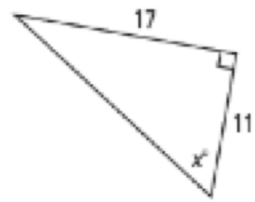
6.4 Primary Trig Ratios

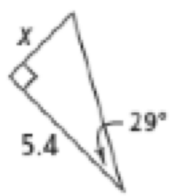
- SOHCAHTOA

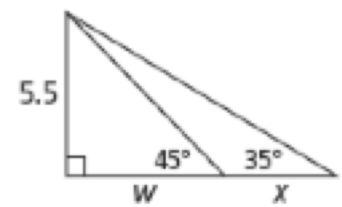


6.4 Primary Trig Ratios - Practice

1. 

7. 

3. 

9. 

5. An escalator at a shopping center is 200 ft 9 in. long, and rises at an angle of 15°. What is the vertical rise of the escalator? Round to the nearest inch.

11. A right triangle has a hypotenuse of length 10 and one leg of length 7. Find the length of the other leg and the measures of the acute angles in the triangle. Round your answers to the nearest tenth

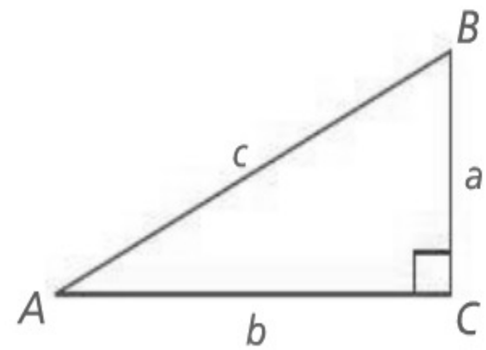
6.5 Reciprocal Trig Ratios

- *Obj.:* I will be able to identify reciprocal trigonometric ratios and use them to find side lengths in a right triangle.

- Vocabulary

○ Reciprocal Trigonometric Ratios	○ Cosecant
○ Secant	○ Cotangent

6.5 Reciprocal Trig Ratios



- Reciprocal Trigonometric Ratios
  - Used to find lengths of sides in a right triangle.
  - These are derived by flipping each of the trigonometric ratios.

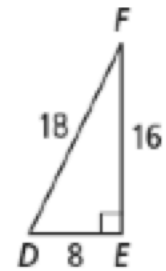
$$\text{Cosecant: } \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{hypotenuse}{opposite} \qquad \csc(\mathrm{A}) = \frac{c}{a}$$

$$\text{Secant: } \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{hypotenuse}{adjacent} \qquad \sec(\mathrm{A}) = \frac{c}{b}$$

$$\text{Cotangent: } \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{adjacent}{opposite} \qquad \cot(\mathrm{A}) = \frac{b}{a}$$

6.5 Reciprocal Trig Ratios - Practice

Use Δ*DEF* and the definitions below to write each ratio.

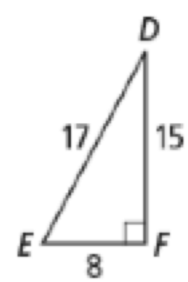


1. csc *D*
2. sec *D*
3. cot *D*
4. csc *F*
5. sec *F*
6. cot *F*

7. An *identity* is an equation that is true for all the allowed values of the variable. Use what you know about trigonometric ratios to show that the equation

   $\cot X = \dfrac{\cos X}{\sin X}$ is an identity.

Use Δ*DEF* and the definitions below to write each ratio.



8. csc *D*
9. sec *D*
10. cot *D*

6.6 Elevation

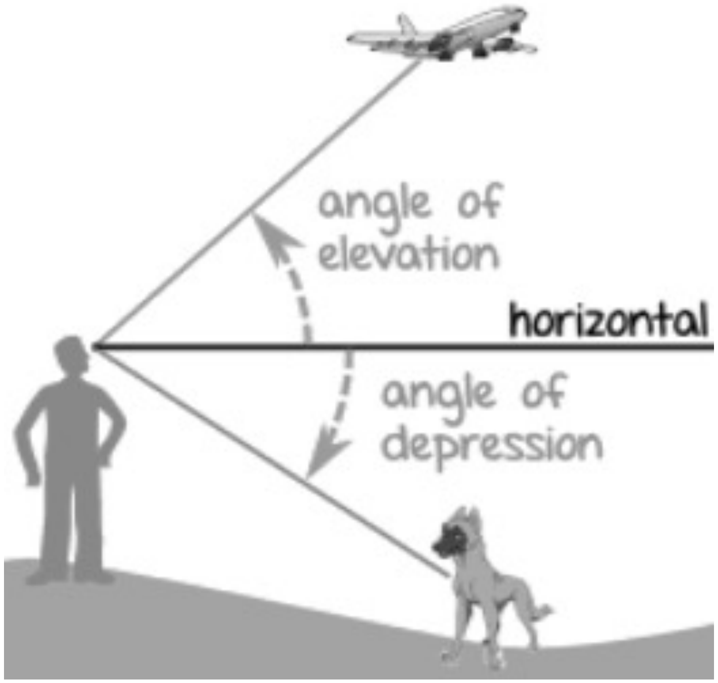
- *Obj.:* I will be able to identify angles of elevation and depression. I will be able to use these angles to indirectly measure distances.

- Vocabulary

○ Angle of Elevation	○ Angle of Depression

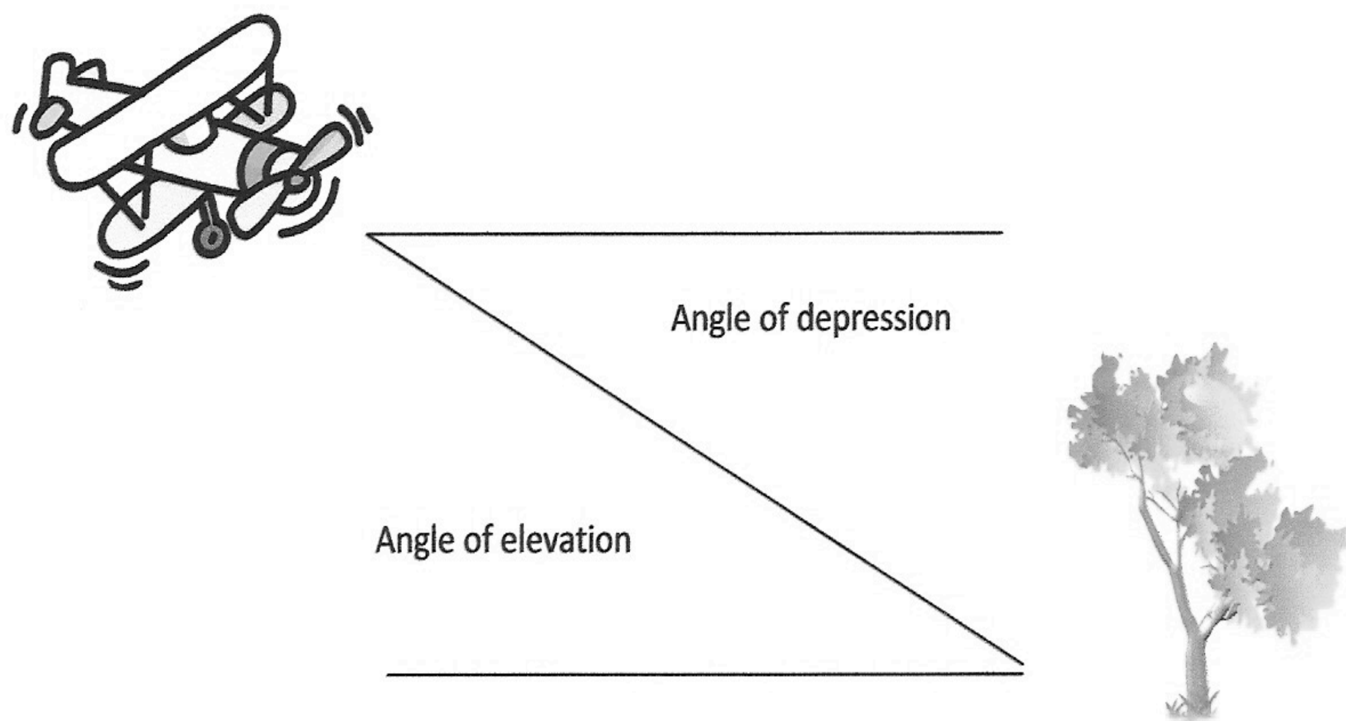
6.6 Elevation

- Angles of Elevation and Depression
  - Elevation – The angle above a horizontal reference
  - Depression – The angle below a horizontal reference



6.6 Elevation

- For a single line of sight, these two angles are congruent because they are alternate interior angles.



6.6 Elevation - Practice

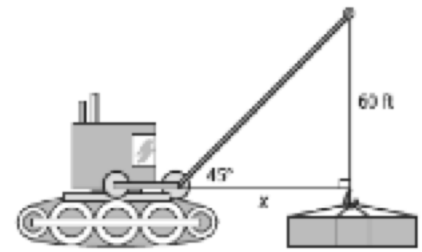
1. $\angle 1$

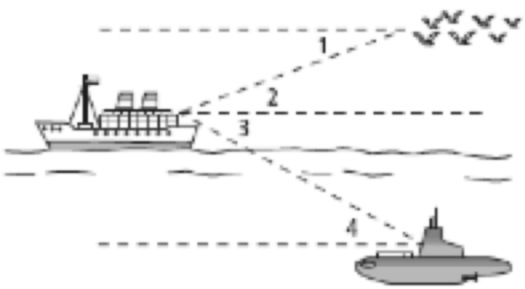
2. $\angle 2$

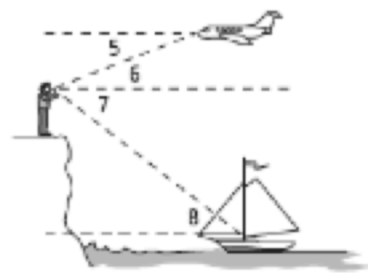
3. $\angle 5$

4. $\angle 6$

5.







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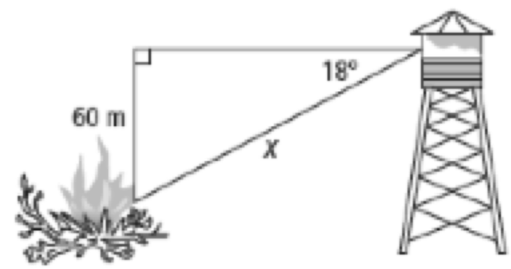

9. A person is standing 40 ft from a flagpole and can see the top of the pole at a 35° angle of elevation. The person's eye level is 4 ft from the ground. What is the height of the flagpole to the nearest foot?

11. *e*: $(3x + 6)°$, *d*: $(x + 20)°$