Unit 1 - Transformations

M1 CH 8.1-4, M2 CH 6.6

1.1 Bellringer – due in 10 min

1. Solve: $2(7v - 3) + 7v = 120$
2. Solve: $-2(1 + 3n) = -20 - 3n$
3. Factor: $x^2 - 11x + 18$
4. Factor: $b^2 + 11b + 24$

Copy down each problem and show all work. Submit to the box at the front with your period marked on it.

1.1 Dilations
Objective and Vocabulary

- *Obj.:* I will be able to identify and write rules for dilations. I will be able to dilate a shape about a given point.
- Vocabulary

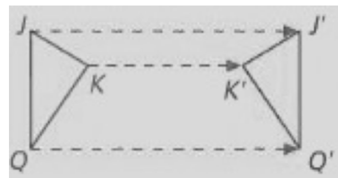
o **Dilation**	o **Center of Dilation**	o **Scale Factors**
o **Enlargement**	o **Reductions**	o Transformation
o Translation	o Preimage	o Image

1.1 Dilations

- Transformations – changes in position, shape, or size of a figure.
  - Flips, slides, and turns are transformations that keep shape and size the same
    - Called rigid motions
    - Only change position
  - Original figure is called the preimage
  - Transformed figure is called the image

1.1 Dilations

- All points of the preimage are mapped into the image.
- 2 notations
  - Arrow notation (→): $\Delta JKQ \rightarrow \Delta JKQ$
  - Prime notation (‘): ΔJKQ maps onto $\Delta J'K'Q'$
- Order matters in named images and preimages.
  - Corresponding parts are listed in the same order.
  - Ex: If $\Delta NID \rightarrow \Delta SUP$, the image of $\angle N$ is $\angle S$



1.1 Dilations

- A Dilation makes a figure bigger or smaller (NOT A RIGID MOTION!)
  - Center of Dilation is the point around which the dilation occurs.
    - The image of the center of dilation is itself.

1.1 Dilations

- Scale Factor (n) indicates how the size will change (n is positive)
  - Enlargement (gets bigger) $n > 1$
  - Reduction (gets smaller) $0 < n < 1$
  - Calculated by $n = image\ length / preimage\ length$ for corresponding parts

1.1 Dilations

- Notation
  - Around the origin
    - $D_n(x,y) = (nx, ny)$
  - Around other points
    - $D_{(n,C)}(DEF) = D'E'F'$ where C is the center of dilation
- Dilations and scale factors can be used to understand real-world enlargements and reductions, such as seeing images through a microscope or on a computer screen.

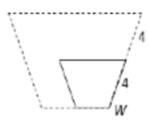
1.1 Dilations – Practice

In each diagram, the dashed-line figure is an image of the solid-line figure. (a) Choose an angle or point from the preimage and name its image. (b) List all pairs of corresponding sides.

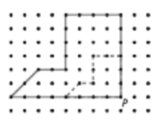
1.

The solid-line figure is a dilation of the dashed-line figure. The labeled point is the center of dilation. Tell whether the dilation is an enlargement or a reduction. Then find the scale factor of the dilation.

3.



5.



You look at each object described in Exercises 7–8 under a magnifying glass. Find the actual dimension of each object.

7. The image of a ribbon is 10 times the ribbon's actual size and has a width of 1 cm.

A dilation has center (0, 0). Find the image of each point for the given scale factor.

9. $X(3, 4)$; $D_7(X)$

11. $M(2, 2)$; $D_5(M)$

1.1 Dilations – Assignment

- Complete 1.1 Dilations on MathXLforschool.com

1.2 Translations
Objective and Vocabulary

- *Obj.:* I will be able to identify and write rules for geometric translations. I will be able to translate a shape in the coordinate plane.
- Vocabulary

o Rigid motion	o Composition of transformations	

1.2 Translations

- Translations slide figures. (RIGID MOTION)
  - All points are translated the same distance in the same direction.
  - Sides maintain their lengths.
  - Angles stay the same measure.
  - Notations
    - Function notation: $T\text{-}\downarrow<h,k>\ (ABCD)=A'B'C'D'$
      - h is horizontal translation (neg = left, pos = right)
      - k is vertical translation (neg = down, pos = up)
    - Coordinate notation: $(x,y)\rightarrow(x+h,\ y+k)$

1.2 Translations

- Writing Translation Rules
  - Use function notation
  - Pick one point and find its image.
  - Count how many spaces horizontally the point move (h). Repeat for vertical distance (k).
  - Fill in function notation with h and k and the name of the preimage and image.

1.2 Translations

- Composition of transformations
  - Two or more translations of a figure result in a different translation of a figure.
  - To find what the overall translation was, add the h values and k values.
  - Ex: If $T_{<2,4>}(ABCD)=EFJK$ and $T_{<-5,1>}(EFJK)=MNPQ$, then $T_{<-3,5>}(ABCD)=MNPQ$

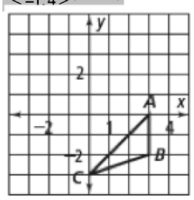
1.2 Translations Practice

Tell whether the transformation appears to be a rigid motion. Explain.

1.

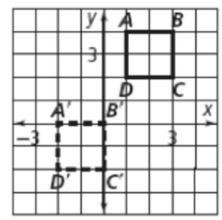
Graph the image of each figure under the given translation.

3. $T_{<-1,4>}(\Delta ABC)$



The dashed-line figure is a translation image of the solid-line figure. Write a rule to describe each translation.

5.



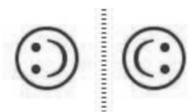
7. You are visiting Washington, D.C. From the American History Museum you walk 5 blocks east and 1 block south to the Air and Space Museum. Then you walk 8 blocks west to the Washington Monument. Where is the Washington Monument in relation to the American History Museum?

1.2 Translations - Assignment

- Complete 1.2 Translations on MathXLforschool.com

1.3 Bellringer – due in 10 min

Copy down each problem and show all work. Submit to the box at the front with your period marked on it.

1. Does the image show a rigid motion? Explain. 
2. Give the rule to translate ABCD 3 spaces left and 2 spaces up.
3. If A(2,3), B(2,6), & C(5,4), and $T_{<-1,-3>}(\Delta ABC)$ what are the coordinates of A', B', & C'?
4. Write the rule for the translation of DEFG directly to LMNO:

$T_{<2,5>}(DEFG)=HIJK$ and $T_{<-8,2>}(HIJK)=LMNO$

1.3 Reflections Objective and Vocabulary

- *Obj.:* I will be able to identify and write rules for reflections. I will be able to reflect a shape across a given line.
- Vocabulary

○ Reflection	○ Line of Reflection	○ Perpendicular Bisector

1.3 Reflections

- A reflection is a flip of a figure. (RIGID MOTION)
  - Reflected figures have opposite orientations
  - Line of reflection:
    - The line over which the figure was reflected.
    - The image of any point on the line of reflection is itself (it doesn't move)

1.3 Reflections

- Perpendicular bisector of the line between corresponding points of the image and preimage
  - Cuts the line in half
  - Intersects the line at a right angle

1.3 Reflections

- Notation
  - Function Notation:
    - $R_m(P)=P'$ where *m* is the line of reflection
    - The line of reflection may be written as an equation.
  - Coordinate Notation:
    - $R_{x-axis}: (x,y)\to(x,-y)$
    - $R_{y-axis}: (x,y)\to(-x,y)$

1.3 Reflections

- Writing Reflection Rules
  - Try connecting corresponding points between two image to see if the line of reflection is a perpendicular bisector. If not, the images are not reflections of one another over that line.
  - Remember that an image can be reflected back over the same line of reflection onto its preimage.

1.3 Reflections – Honors Only

- Reflections over other lines
  - Horizontal (y = #): Relate the location of the preimage to the line of reflection (how far above or below it is), and plot that many spaces on the other side. The x-value stays the same.
  - Vertical (x = #): Relate the location of the preimage to the line of reflection (how far left or right it is), and plot that many spaces on the other side. The y-value stays the same.

1.3 Reflections – Honors Only

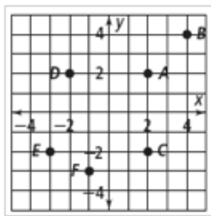
- Diagonal Lines
  - $R\downarrow y=x$: $(x,y)\rightarrow(y,x)$
  - $R\downarrow y=-x$: $(x,y)\rightarrow(-y,-x)$

1.3 Reflections – Practice

Find the coordinates of each image.

1. $R_{x\text{-}axis}(A)$

3. $R_{y=1}(C)$



Coordinate Geometry Given points $\underline{M}(3, 3)$, $N(5, 2)$, and $O(4, 4)$, graph ΔMNO and its reflection image as indicated.

5. R_{y-axis}

Copy each figure and line *l*. Draw each figure's reflection image across line *l*.

7.

9. Point *A* on a coordinate grid is at (3, 4). What are the coordinates of $R_{y=x}(A)$?

1.3 Reflections - Assignment

- Complete 1.3 Reflections on MathXLforschool.com

1.4 Bellringer – due in 10 min

1. $R_{y\text{-axis}}$(ABC); A(−4, −1), B(−4, 1), C(0, −3)
2. $R_{y\text{-axis}}$(FGH); F(−4, −3), G(−5, 2), H(−1, −2)
3. $R_{x\text{-axis}}$(UTS); U(−2, −4), T(−3, 0), S(1, −1)
4. $R_{x\text{-axis}}$(RST); R(3, −3), S(2, 1), T(4, 2)

Copy down each problem and show all work. Submit to the box at the front with your period marked on it. Find the coordinates of the image for each of the indicated reflections.

1.4 Rotations
Objective and Vocabulary

- *Obj.:* I will be able to identify and write rules for rotations. I will be able to rotate a shape about a given point.
- Vocabulary

○ Rotation	○ Center of Rotation	○ Angle of Rotation

1.4 Rotations

- A rotation is a turn of a figure. (RIGID MOTION)
  - The point about which a figure is rotated is called the center of rotation.
  - The center of rotation does not move.

1.4 Rotations

- The number of degrees a figure rotates is the angle of rotation
  - The angle between the preimage and the image with the center of rotation at the vertex.
  - Figures are rotated counterclockwise unless otherwise stated.

1.4 Rotations

- Notation
  - Function Notation: $r_{(x°,Q)}(\Delta UVW) = \Delta U'V'W'$
    - x is the angle of rotation
    - Q is the center of rotation
  - Coordinate Notation:
    - $r_{90°,O}$: $(x,y) \to (-y,x)$
    - $r_{180°,O}$: $(x,y) \to (-x,-y)$
    - $r_{270°,O}$: $(x,y) \to (y,-x)$
    - $r_{360°,O}$: $(x,y) \to (x,y)$

1.4 Rotations – Honors Only

- Rotation about other points
  - Subtract the center of rotation from each of the vertex coordinates. (Translate the shape so that the center of rotation is the origin)
  - Rotate using the rule above for each point.
  - Add the center of rotation back to each point. (Translate the figure back for the correct center of rotation)

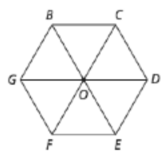
1.4 Rotations – Practice

Point *O* is the center of regular hexagon *BCDEFG*. Find the image of the given point or segment for the given rotation.

1. $r_{(120^\circ, O)}(F)$

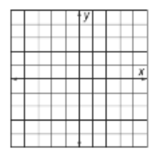
3. $r_{(300^\circ, O)}(\overline{BG})$

5. $r_{(60^\circ, O)}(E)$



For Exercises 7–8, $\triangle ABC$ has vertices *A*(2, 2), *B*(3, −2), and *C*(−1, 3).

7. Graph $r_{(180^\circ, O)}(\triangle ABC)$.



9. The vertices of *PQRS* have coordinates *P*(-1, 5), *Q*(3, 4), *R*(2, -4), and *S*(−3, −2). What are the coordinates of the vertices of $r_{(270^\circ, O)}(PQRS)$?

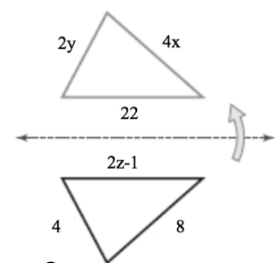
1.4 Rotations - Assignment

- Complete 1.4 Rotations on MathXLforschool.com

1.5 Bellringer – due in 10 min

Copy down each problem and show all work

1. For the given points, A'B' is a translation of AB. Write a rule to describe the translation. A (–6, 1), B(5, 3), A'(–5, 10), B'(6,12)
2. Point C(x, y) moves 4 units right and 9 units down. Write a rule in coordinate notation. (x, y) → ?
3. Find the value of each variable in the diagram.
4. Solve the equation by finding square roots: $x^2 - 16 = 0$



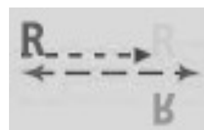
1.5 Compositions of Isometries Objective and Vocabulary

- *Obj.:* I will be able to recognize and perform a composition of isometries and identify the individual isometries. I will be able to perform a glide reflection for a figure.
- Vocabulary

o Glide Reflection	o Isometry	

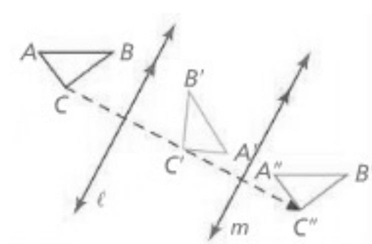
1.5 Compositions of Isometries

- An isometry is a transformation that preserves distance, or length.
- Rigid motions are isometries
- All isometries are compositions of reflections.
- Four kinds of isometries
  - Translation
  - Rotation
  - Reflection
  - Glide Reflection



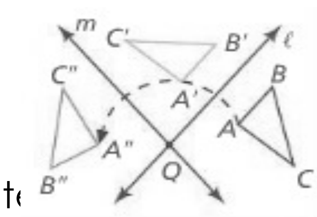
1.5 Compositions of Isometries

- The composition of two or more isometries is an isometry.
  - A composition of reflections across two parallel lines is a translation.
    - 2 ways to write the composition:
      - $(R\downarrow m \circ R\downarrow l)(\Delta ABC) = \Delta A''B''C''$
      - $R\downarrow m\,(R\downarrow l\,(\Delta ABC)) = \Delta A''B''C''$



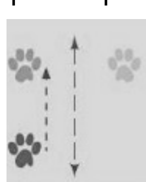
1.5 Compositions of Isometries

- A composition of reflections across two intersecting lines is a rotation
  - 2 ways to write
    - $(R_m \circ R_l)(\Delta ABC)=\Delta A''B''C''$
    - $R_m(R_l(\Delta ABC))=\Delta A''B''C''$
  - The figure is rotated about the int wo lines.
  - The angle of rotation is equal to twice the angle between the two lines through which the figure was reflected.



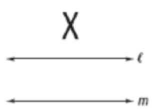
1.5 Compositions of Isometries

- Glide Reflection
  - Composition of a translation (glide) and a reflection across a line parallel to the translation.
  - Notation: $(R_m \circ T_{<h,k>})(\Delta ABC)=\Delta A''B''C''$
  - You can map a left paw print onto a right paw print using glide reflection.



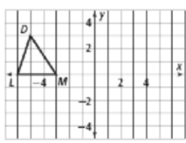
1.5 Compositions of Isometries Practice

Find the image of each letter after the transformation $R_m \circ R_\ell$. Is the resulting transformation a translation or a rotation? For a translation, describe the distance and direction. For a rotation, tell the center of rotation and the angle of rotation.

1. 

Graph ΔDML and its glide reflection image.

3. $(R_{y\text{-axis}} \circ T_{<3, 0>})(\Delta DML)$



5. Lines *l* and *m* intersect at point *P* and are perpendicular. If a point *Q* is reflected across *l* and then across *m*, what transformation rule describes this composition?

Graph $\overline{AB}$ and its image $\overline{A'B'}$ after a reflection first across l_1 and then across l_2. Is the resulting transformation a translation or a rotation? For a translation, describe the direction and distance. For a rotation, tell the center of rotation and the angle of rotation.

7. $A(-3, 4)$, $B(-1, 0)$;
   l_1 : *x* - axis;
   l_2 : *y* - axis

1.5 Compositions of Isometries Assignment

- ▶Complete 1.5 Compositions of Isometries on MathXLforschool.com
- ▶Complete Review 1