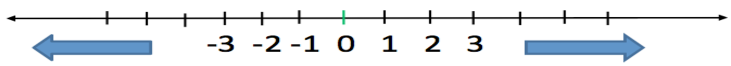
Unit 1 – Linear Expressions

1.1 Operations with Negative Numbers

- Order of Operations (P E MD AS)
    - Grouping/Parentheses
    - Exponents
    - Multiplication/Division (in order as written)
    - Addition/Subtraction (in order as written)
- Real numbers include numbers on both sides of the number line. We need to be able to perform arithmetic operations on al real numbers, both positive and negative.
- On the number line, values increase towards the right and decrease towards the left.



- Addition
    - When adding a positive number, move right on the number line.
        - Examples: 3 + 2 = 5 and -4 + 1 = -3
    - When adding a negative number, move left on the number line.
        - Examples: 3 + -2 = -1 and -7 + -4 = -11
    - General rules:
        - When adding two values with the same sign, add the absolute value of each number and keep the sign.
        - When adding two values with different signs, find the difference of the absolute values of each number and keep the sign of the addend with the greater absolute value.
- Subtraction
    - Can be restated as addition (adding the inverse)
    - Examples: -3 – 2 = -3 + -2 = -5 and 2 – 6 = 2 + 6 = -4
- Multiplication and division
    - When multiplying/dividing two values with the same sign, the product is always positive.
        - Examples of Multiplication: (3)(4) = 12 and (-5)(-3) = 15
        - Examples of Division: 14/7 = 2 and -18/-2 = 9
    - When multiplying/dividing two values with different signs, the product is always negative
        - Examples of Multiplication: (3)(-6) = -18 and (-9)(4) = -36
        - Examples of Division: -16/4 = -4 and 108/-12 = -9
    - When multiplying many factors to find a product, multiply two values at a time.
- Exponents
    - When a negative number is raised to an exponent, one of two cases occurs:
        - If the exponent is odd, the answer will be negative [Ex: $(-3)^3 = -27$]
        - If the exponent is even, the answer will be positive [Ex: $(-3)^2 = 9$]

1.2 Operations with Fractions

- Fractions are just division! $\frac{numerator}{denominator}$
- Simplifying Fractions
  - A fraction can be simplified if both the numerator (top number) and the denominator (bottom number) have a common factor.
  - Example: $\frac{15}{10}$ → both 15 and 10 are divisible by 5 (GCF). $\frac{15 \div 5}{10 \div 5} = \frac{3}{2}$
  - All answers given as fractions should be given in the simplest form.
- Multiplication
  - When multiplying fractions, multiply the numerators together to get the numerator of the product and the denominators together to get the denominator of the product.
  - Then reduce the fraction if necessary.
  - Formula: $\frac{a}{b} * \frac{c}{d} = \frac{ab}{cd}$
  - Example: $\frac{3}{4} * \frac{1}{5} = \frac{3*1}{4*5} = \frac{3}{20}$
- Division
  - Division is the inverse of multiplication. In order to divide fractions, change them to multiplication of the reciprocal.
  - Follow the multiplication procedure from this point. Reduce if necessary.
  - Formula: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} * \frac{d}{c} = \frac{ad}{bc}$
  - Example: $\frac{8}{11} \div \frac{1}{2} = \frac{8}{11} * \frac{2}{1} = \frac{8*2}{11*1} = \frac{16}{11}$
- Addition/Subtractions
  - Like Fractions
    - When fractions have the same denominator, they are called "like fractions" and can be easily added/subtracted.
    - Add/subtract the numerators and keep the denominator. Reduce.
    - Formula: $\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$
    - Examples: $\frac{3}{8} + \frac{1}{8} = \frac{3+1}{8} = \frac{4}{8} = \frac{1}{2}$ and $\frac{15}{16} - \frac{4}{16} = \frac{15-4}{16} = \frac{11}{16}$
  - Unlike Fractions
    - When fractions do not have the same denominator, they are called "unlike fractions."
    - To add/subtract, they must be converted to "like fractions."
    - Finding the ECD (easiest common denominator) → multiply the denominators together.
    - To convert two fractions in the same expression: multiply each fraction by the other fraction's denominator over itself.
    - Then follow rules with "like fractions."
      - Formula: $\frac{a}{b} \pm \frac{c}{d} = \left(\frac{d}{d}\right)\left(\frac{a}{b}\right) \pm \left(\frac{c}{d}\right)\left(\frac{b}{b}\right) = \frac{ad \pm bc}{bd}$
      - Example: $\frac{6}{7} + \frac{4}{5} = \left(\frac{5}{5}\right)\left(\frac{6}{7}\right) + \left(\frac{4}{5}\right)\left(\frac{7}{7}\right) = \frac{30}{35} + \frac{28}{35} = \frac{58}{35}$
- Converting Mixed Numbers to Improper Fractions

- Mix number consists of a whole number and a fraction. (Ex: $1\frac{1}{2}$)
- To convert from a mixed number to an improper fraction, multiply the whole number by the denominator over itself, then add the two fractions together.
  - Example: Rewrite $7\frac{3}{4}$ as an improper fraction
  - The denominator = 4. convert $\left(\frac{4}{4}\right)\left(\frac{7}{1}\right) = \frac{28}{4}$ then add: $\frac{28}{4} + \frac{3}{4} = \frac{31}{4}$

1.3 Conversions Using Dimensional Analysis

- Dimensional analysis is the method of converting different units of measurement, such as cm to inches, feet to meters, kilometers to meters, etc.
- Conversion factors are fractions (=1) that can be made from equalities relating two units.
  - Examples: 2.54cm = 1 in. can be written as $\frac{2.54cm}{1in}$ or $\frac{1in}{2.54cm}$

    32°F = 273.15K can be written as $\frac{32°F}{273.15K}$ or $\frac{273.15K}{32°F}$
- Conversion factors allow for measurements in one unit to be expressed equivalently in another unit.
- If a unit is raised to an exponent, the conversion factor between it and another unit can be raised to that exponent to relate them.
  - Example: if 1 m = 3.28 ft, then $\left(\frac{1m}{3.28ft}\right)^3 = \frac{(1m)^3}{(3.28ft)^3} = \frac{1m^3}{35.29ft^3}$
- Using a grid can help to see which form of the conversion factor you will need.

  is the same as multiplying two fractions $(\frac{-}{-} * \frac{-}{-})$
- Always start with the value to be converted.
- Place conversion factors into the grid such that units will cancel out (remember that anything divided by itself = 1, including units!)
  - Example: Convert 25 inches to centimeters.

25 ~~inches~~	2.54 cm
	1 ~~inch~~

    $=\left(\frac{25}{1}\right)\left(\frac{2.54}{1}\right)$ = 63.5 cm
- Multiple conversions can be done in this format at the same time.
  - Make sure to cancel out units as you go to keep track of what you have.
  - Example: Convert 10 years to seconds.

10 ~~years~~	365.25 ~~days~~	24 ~~hours~~	60 ~~min~~	60 seconds
	1 ~~year~~	1 ~~day~~	1 ~~hour~~	1 ~~min~~

=315576000 seconds

- Conversions can also be done for units in the denominator or both numerator and denominator.
  - Example: Convert 15 g/mL to g/L

15g	1000~~mL~~
~~mL~~	1L

    = 15000 g/L

  Example: Convert 17kg/mL to g/L

17 ~~kg~~	1000g	1000 ~~mL~~
~~mL~~	1 ~~kg~~	1 L

  = 17000000 g/L

1.4 Dissecting Terms in Algebraic Expressions

- Order of operations
  - P
  - E – exponents/powers
  - MD – factors (mult./div together)
  - AS – terms (Separated by +/-)
- Expression – a set of terms
- Equation – 2 expressions are equal
  - Ex: $4x - 7 = 5$
    - Terms: 4x, -7, 5
    - Expressions: 4x – 7, 5
- Coefficient - # multiplied by a variable
- Constant - # on its own
- Degree (order) – highest sum of exponents in a term (order of expression is the highest order of any term in that expression)
- Polynomials:
  - Monomial (1term) → 5x (1st degree polynomial)
  - Binomial (2 terms) → $3x + 6y^4$ (4th degree polynomial)
  - Trinomial (3 terms) → 2x + 7xy + 8y (2nd degree polynomial)
- Lead coefficient – coefficient of highest degree term in expression
  - For each of the polynomials above, the lead coefficients are: 5, 6, 7

1.5 Solving Linear Equations

- Combining like terms
  - must have same variable and exponent (add/subtract only coefficients)
  - ex: 3n + 4n = 7n but $7n^2 + 8n \neq 15n^2$ or $15n^3$
- Solving a multi-step problem
  - Set up an equation (if none given) based on the info provided
  - Use distributive property as needed [ex: 8(n+ 3) = 8n + 24]
  - Combine like terms
  - Use the reverse order of operations to isolate the variable
- Fractions and decimals
  - If fractions are involved, convert to like fractions. Multiply equation by new denominator.
  - If decimals are involved, multiply equation (both sides) by 10^x where x = number of decimal places in longest decimal.
- Some equations have no solution (if x cancels out entirely on both sides and you are left with a false statement such as -4 = 5).
- Some equations have infinite answers (if you are left with the same thing on both sides when you have simplified the equation → 6 = 6)

1.6 Literal Equations and Formulas

- Literal equation → involved two or more variables
- Just as when solving multi-step equations, you can rearrange a literal equation for just one variable.

- Ex: rearrange 14x + 7y = 21 for y.

$$7y = 21 - 14x$$
$$y = 3 - 2x$$

- Ex: rearrange ax – bx = c for x.

$$x(a - b) = c$$
$$x = \frac{c}{a - b}$$

- Given a value for one variable, substitute the value in place of the variable and solve.
  - Ex. Solve 3x + 7y = 23 when x = 3

$$3(3) + 7y = 23$$
$$9 + 7y = 23$$
$$7y = 14$$
$$y = 2$$

- Formula – an equation that states the relationship between two or more quantities. (Ex: C = 2πr)
  - Can re-write formulas to solve for another variable
  - Ex: C = 2πr, solve for r

$$C = 2\pi r$$
$$\frac{C}{2\pi} = \frac{2\pi r}{(2\pi)}$$
$$\frac{C}{2\pi} = r$$

1.7 Linear Inequalities

- Solved in the same way as a multi-step equation
  - Major difference in procedure: if you multiply or divide both sides by a negative number, reverse the inequality sign.
  - Ex: 9 + 4t > 21

$$4t > 12$$
$$t > 3$$

- Formulas can be adapted for inequalities, as well (ex: p. 61 student book)

Geometry In a community garden, you want to fence in a vegetable garden that is adjacent to your friend's garden. You have at most 42 ft of fence. What are the possible lengths of your garden?

$$P = 2l + 2w$$
$$42 \geq 2l + 2(12)$$
$$42 \geq 2l + 24$$
$$18 \geq 2l$$
$$9 \geq l$$

- Additional examples
  - 3(t + 1) – 4t ≥ -5

$$3t + 3 - 4t \geq -5$$
$$3 - t \geq -5$$

$-t \geq -8$
$t \leq 8$

- 6n -1 > 3n + 8
  $3n - 1 > 8$
  $3n > 9$
  $n > 3$

- Just like equations, some inequalities can be never true or always true (identities).
  - Ex: $10 - 8a \geq 2(5 - 4a)$
    $10 - 8a \geq 10 - 8a$
    $10 \geq 10$ ALWAYS TRUE

    $6m - 5 > 7m + 7 - m$
    $6m - 5 > 6m + 7$
    $-5 > 7$ NEVER TRUE
- Graphing answers to inequalities on a number line.
  - Use open dots if > or <
  - Use closed dots if ≥ or ≤
  - If the variable is > or ≥, shade to the right. If the variable is < or ≤, shade to the left.
  - Ex: graph each of the above.