UNIT 4 SIMILARITY AND CONGRUENCE

M2 Ch. 2, 3, 4, 6 and M1 Ch. 13

4.1 Parallel Lines

- Objective
  - When parallel lines are cut by a transversal, I will be able t identify angle relationships, determine whether angles are congruent, supplementary, or both, and combine the theorems/postulates with algebra to solve for angle meas
- Vocabulary

o Same-Side Interior Angles Postulate	o Alternate Interior Angles Postulate
o Alternate Exterior Angles Postulate	o Corresponding Angles Postulate

4.1 Parallel Lines - Extras

- Same-side interior angles: angles on the same side of the transversal inside the parallel lines
- Alternate Interior Angles: Angles on opposite sides of the transversal, inside the parallel lines
- Corresponding Angles: Angles on the same side of the transve on different intersections, one inside, one outside the parallel
- Alternate Exterior Angles: Angles on opposite sides of the transversal and are outside the parallel lines
- Vertical Angles: Angles that share a vertex and are opposite

- Vertical Angles Theorem: Vertical angles are congruent

4.1 Parallel Lines

- Identifying Angle Relationships
  - The special angle pairs formed by parallel lines and a transversal are congruent, supplementary, or both.
    - Supplementary (sum of two angles = 180°):
      - Same-Side Interior Angles Postulate
      - If a transversal intersects two parallel lines, then sam side interior angles are supplementary.
      - $m\angle 4+\mathrm{m}\angle 5=180$ and $m\angle 3+\mathrm{m}\angle 6=180$

4.1 Parallel Lines

- Congruent (angles have the same measure):
  - Alternate Interior Angles Theorem
    - If a transversal intersects two parallel lines, then alternate interior angles are congruent.
    - $\angle 4 \cong \angle 6$ and $\angle 3 \cong \angle 5$
  - Corresponding Angles Theorem
    - If a transversal intersects two parallel lines, then corresponding angles are congruent.
    - $\angle 1 \cong \angle 5$, $\angle 4 \cong \angle 8$, $\angle 2 \cong \angle 6$, and $\angle 3 \cong \angle 7$

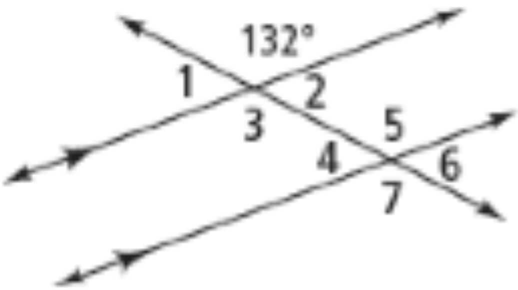
4.1 Parallel Lines

- Alternate Exterior Angles Theorem
  - If a transversal intersects two parallel lines, then alternate exterior angles are congruent.
  - $\angle 1 \cong \angle 7$ and $\angle 2 \cong \angle 8$
- Finding Measures of Angles
  - You can combine theorems and postulates with your know of algebra to find angle measures.

4.1 Parallel Lines - Practice

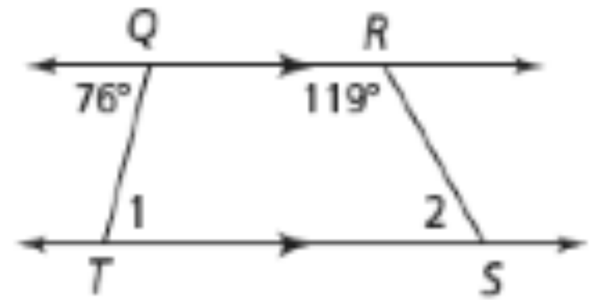
Identify all the numbered angles that are congruent to the given angle. Justify your answers.

1.



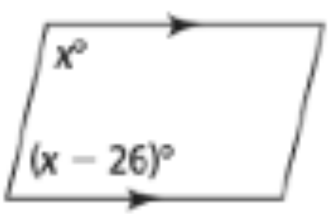
Find $m\angle 1$ and $m\angle 2$ Justify each ans

3.



lgebra Find the value of *x* and *y*. Then find the measure of each labeled angle.

5.



4.2 Congruent Triangles I

- Objective
  - *I will be able to identify congruent figures and correspond parts of congruent figures. I will be able to determine side and angle measure based on congruent figures. I will be a to prove two triangles congruent using SSS, SAS, and congruence transformations*.
- Vocabulary

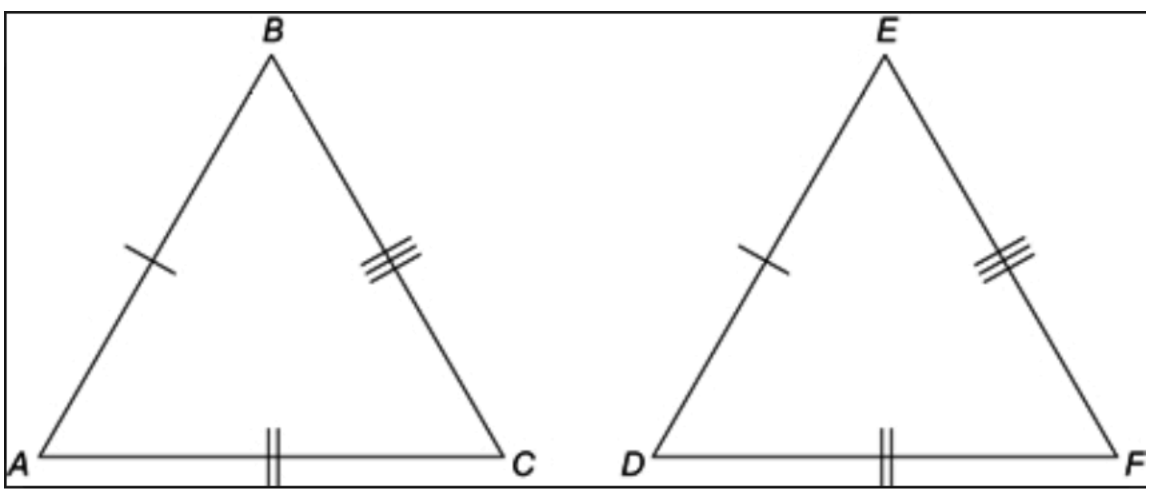
○ Congruent	○ SSS	○ SAS	○ Third Ang Theorem
○ Congruence Transformations		○ Congruent Polygons	

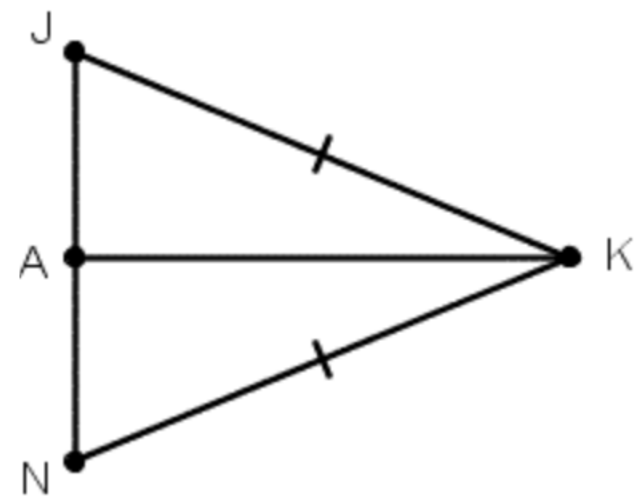
4.2 Congruent Triangles I

- Congruent figures have the same size and shape.
  - Can do compositions of rigid motions to one figure to map onto the other.
  - Congruent polygons have congruent corresponding parts – matching sides and angles.
  - When naming congruent polygons, the corresponding verti must be listed in the SAME ORDER.
- Third Angles Theorem
  - If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.
  - Remember: Angles of any triangle add up to 180°.

4.2 Congruent Triangles I

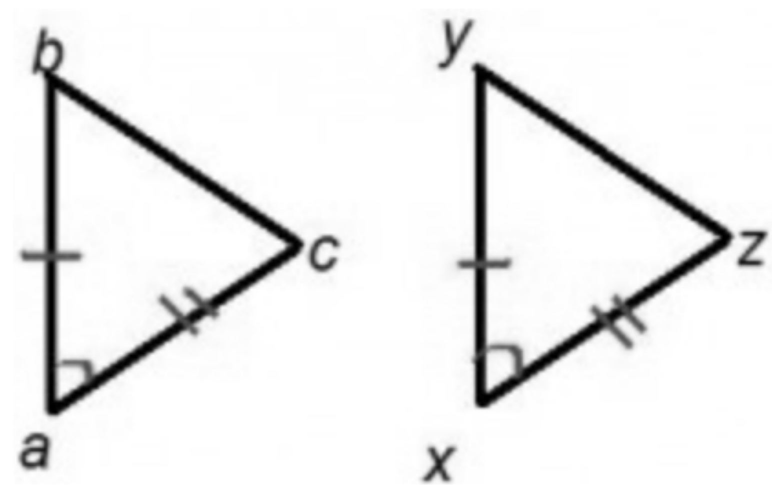
- Triangle Congruence Shortcuts
  - Side-Side-Side (SSS) Postulate
    - If the three sides of one triangle are congruent to the th
      sides of another triangle, then the two triangles are
      congruent.
    - Indicates rigidity of triangles – architects and engineers
      on this!





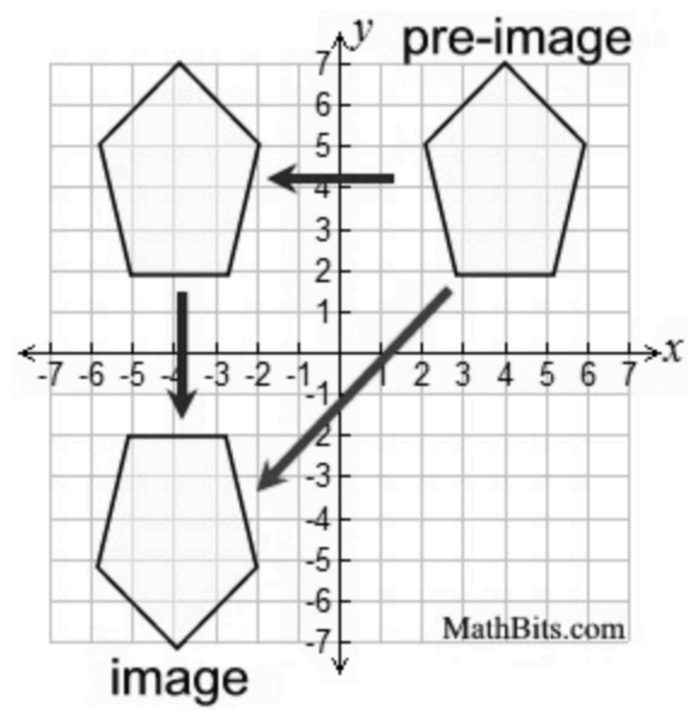
4.2 Congruent Triangles I

- Side-Angle-Side (SAS) Postulate
  - If two sides and the included angle of one triangle are congruent to two sides and the included angle of anoth
triangle, then the two triangles are congruent.
  - Included angle refers to the angle formed by the two sid
as its rays.



4.2 Congruent Triangles I

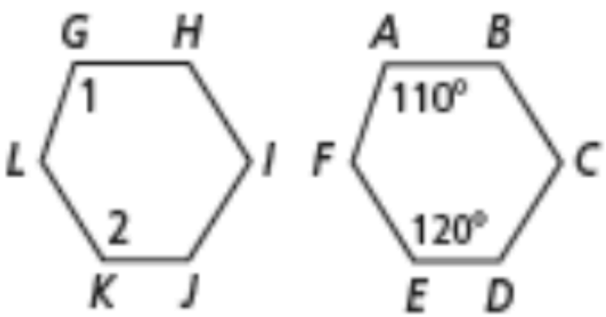
- Congruence Transformations
  - Two figures are congruent if and only if there is a sequence one or more rigid motions that maps one figure onto the ot
  - Compositions of rigid motions that prove congruency are ca congruence transformations.



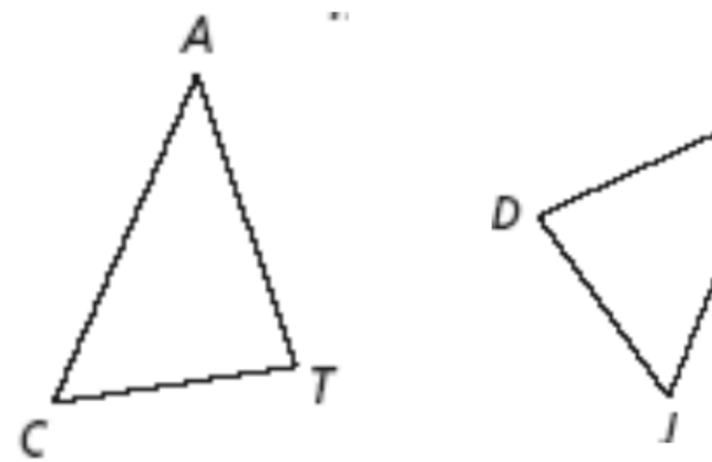
4.2 Congruent Triangles I – Practice

Each pair of polygons is congruent. Find the measures of the numbered angles.

1.



$\Delta CAT \cong \Delta JSD$. List each of the following.

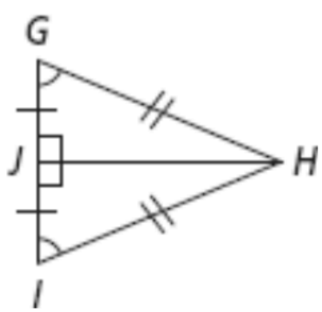


3. three pairs of congruent sides

4.2 Congruent Triangles I – Practice

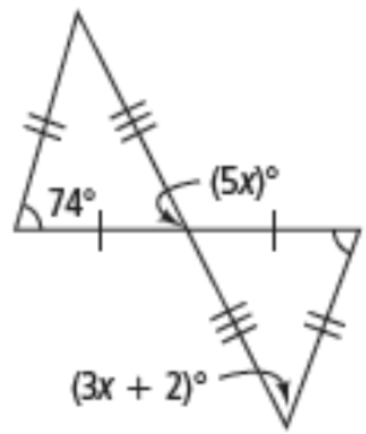
For Exercises 8 and 9, can you conclude that the triangles are congruent? Justify your answers.

5. ΔGHJ and ΔIHJ



Find the values of the variables.

7.



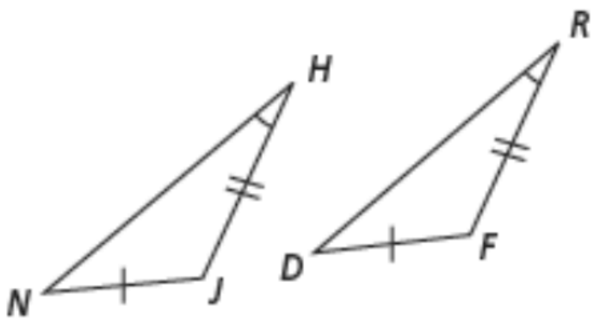
Algebra $ABCD \cong FGHJ$. Find the measures of the given angles or lengths of the given sides.

9. $m\angle B = 3y, m\angle G = y + 50$

4.2 Congruent Triangles I – Practice

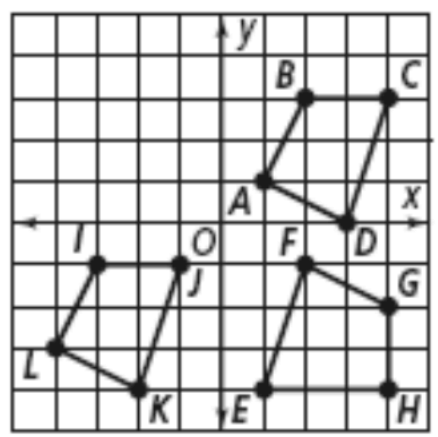
Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information*. Explain your answer.

11.



For each coordinate grid, identify a pair of congruent figures. Then determine a congruence transformation that maps the preimage to the congruent image.

15.



4.2 Congruent Triangles I – Practice

Determine whether the figures are congruent. If so, describe a congruence transformation th
maps one to the other. If not, explain.

17.

4.3 Congruent Triangles II

- Objective
  - I will be able to prove two triangles congruent using ASA, A
    and/or HL. I will be able to identify the hypotenuse and leg
    a right triangle. I will be able to recognize and use the fact
    corresponding parts of congruent triangles are congruent.
- Vocabulary

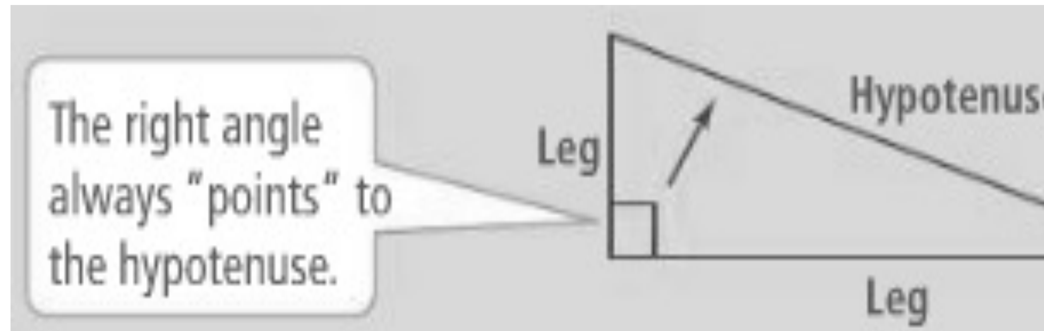
○ ASA	○ AAS	○ CPCTC	○ HL
○ Hypotenuse	○ Legs of a Right Triangle		○

4.3 Congruent Triangles II

- Triangle Congruence Shortcuts
  - Angle-Side-Angle (ASA) Postulate
    - If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.
    - An included side is the shared side of the two angles (between the angles).
  - Angle-Angle-Side (AAS) Theorem
    - If two angles and an nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

4.3 Congruent Triangles II

- Hypotenuse-Leg (HL) Theorem
  - *Anatomy of a right triangle*



  - *Conditions:*
    - Two right triangles
    - Triangles have congruent hypotenuses
    - One pair of congruent legs
- CPCTC - Corresponding Parts of Congruent Triangles are Cong

4.3 Congruent Triangles II - HONORS

- Writing Proofs
  - Three types of proofs:
    - Two-Column
    - Flow
    - Paragraph
  - Proofs use theorems and postulates to support the statem that will get you from the given statement to the prove statement.
  - Use provided diagrams to identify any theorems/postulate might be used.
  - Then, using the theorems/postulates, make statements th provide a logical path from a given to the statement being proved.

4.3 Congruent Triangles II - Practice

ame two triangles that are congruent by ASA.

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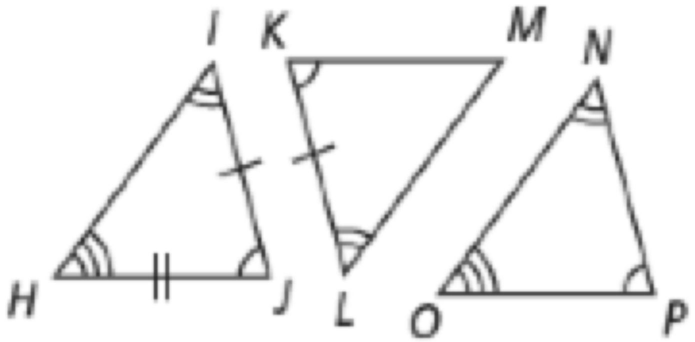
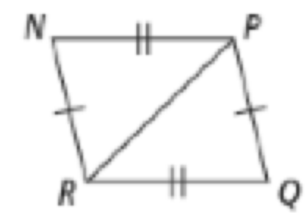


For each pair of triangles, tell why the two triangles are congruent. Give th
statement. Then list all the other corresponding parts of the triangles that a

5.



Developing Proof Complete the proof by filling in the blanks.

Given: $\angle HIJ \cong \angle KIJ$

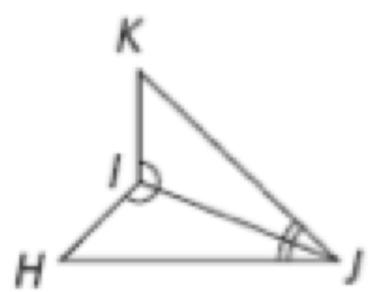
$\angle IJH \cong \angle IJK$

Prove: $\Delta HIJ \cong \Delta KIJ$

Proof: $\angle HIJ \cong \angle KIJ$ and $\angle IJH \cong \angle IJK$ are given.

$\overline{IJ} \cong \overline{IJ}$ by __?__.

So, $\Delta HIJ \cong \Delta KIJ$ by __?__.

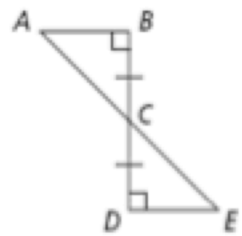


4.3 Congruent Triangles II - Practice

Complete the proof.

Given: $\overline{BD} \perp \overline{AB}, \overline{BD} \perp \overline{DE}, \overline{BC} \cong \overline{DC}$

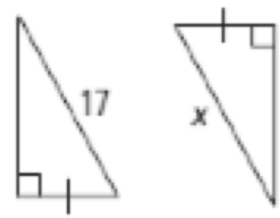
Prove: $\angle A \cong \angle E$



Statements	Reasons
1) $\overline{BD} \perp \overline{AB}, \overline{BD} \perp \overline{DE}$	1) ___?___
2) $\angle CDE$ and $\angle CBA$ are right angles.	2) Definition of right angles.
3) $\angle CDE \cong \angle CBA$	3) ___?___
4) ___?___	4) Vertical Angles are Congruence
5) $\overline{BC} \cong \overline{DC}$	5) ___?___
6) ___?___	6) ___?___
7) $\angle A \cong \angle E$	7) ___?___

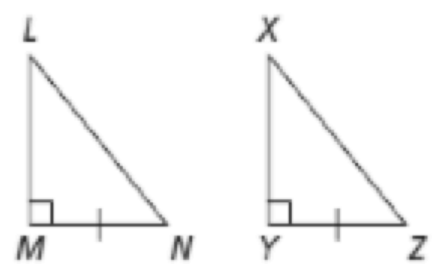
Algebra For what values of x or x and y are the triangles cong

11.



What additional information would prove each pair of triangles congruent by the H
Leg Theorem?

13.



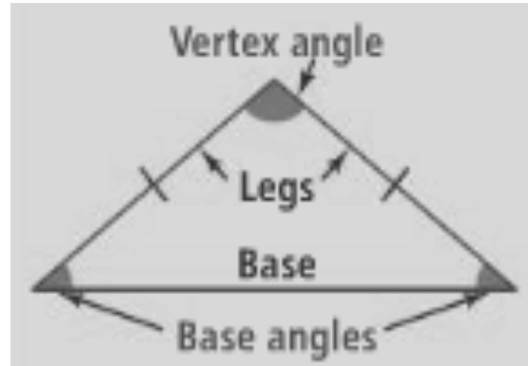
4.4 Isosceles Triangles

- Objective
  - *I will be able to identify parts of an isosceles triangle. I will able to use the theorems and corollaries associated with isosceles triangles to find missing sides, missing angles, a variables.*
- Vocabulary

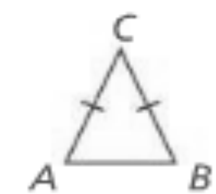
Legs of an isosceles triangle		○ Base of an isosceles triangle
Vertex angle of an isosceles triangle		○ Base angle of an isosceles triangle
Equilateral Triangle		○ Equiangular triangle
Converse of Isosceles Triangle Theorem		○ Isosceles Triangle Theorem
Corollary to Isosceles Triangle Theorem		○ Corollary to Converse of Isosceles Triang Theorem
Corollary	○ Theorem 22	

4.4 Isosceles Triangles

- Anatomy of an Isosceles Triangle



- Theorems
  - *Isosceles Triangle Theorem*
    - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
  - *Converse of the Isosceles Triangle Theorem*
    - If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



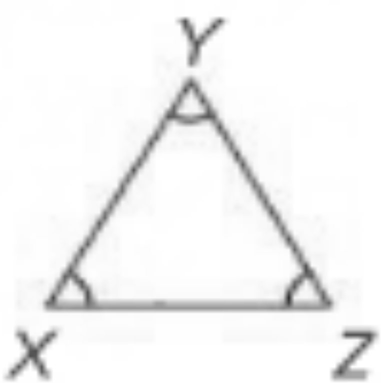
4.4 Isosceles Triangles

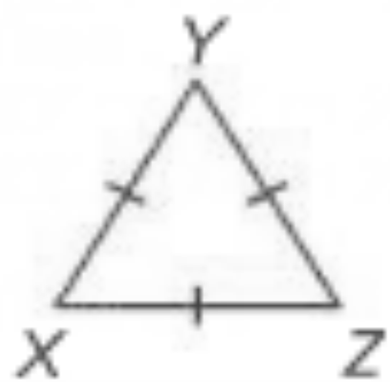
- *Theorem 22*
  - If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.

- Corollaries
  - *A corollary is a theorem that can be proved easily using ar theorem.*
  - *Can be used as a reason in a proof*

4.4 Isosceles Triangles

- *Corollary to the Isosceles Triangle Theorem*
  - If a triangle is equilateral, then the triangle is equiangul
  - Equilateral – all sides are congruent to each other
  - Equiangular – all angles are congruent to each other
- *Corollary to the Converse of the Isosceles Triangle Theorem*
  - If a triangle is equiangular, then the triangle is equilater



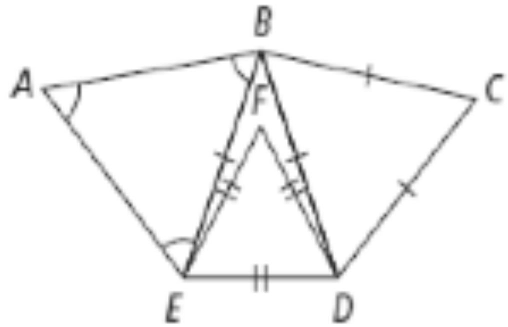


4.4 Isosceles Triangles - Practice

omplete each statement. Explain why it is true.

1. $\angle FED \cong$ __?__ $\cong \angle DFE$

2. $\overline{AB} \cong$ __?__ $\cong \overline{BE}$



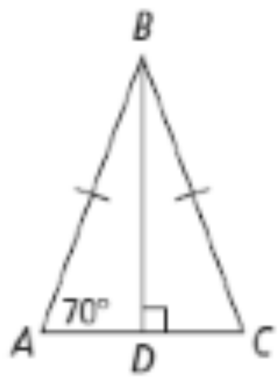
Algebra Find the values of

3.

Jse the properties of isosceles and equilateral triangles to find the measure of the indicated angle.

5. $m\angle ACB$

6. $m\angle DBC$

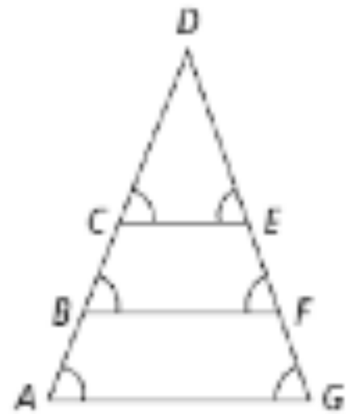


4.4 Isosceles Triangles - Practice

Use the diagram for Exercises 7–8 to complete each congruence statement. Explain why it is true.

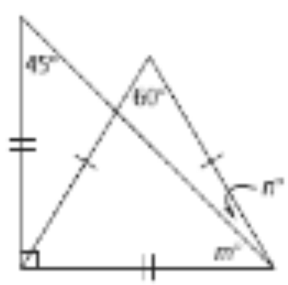
7. $\overline{DF} \cong$ ___?___

8. $\overline{DG} \cong$ ___?___



9. The wall at the front entrance to the Rock and Roll Hall of Fame and Museum in Cleveland, Ohio, is an isosceles triangle. The triangle has a vertex angle of 102. What is the measure of the base angles?

Algebra Find the values of *m* and *n*.

10. 

11. 

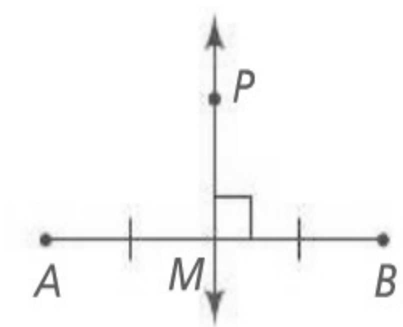
4.5 Bisectors

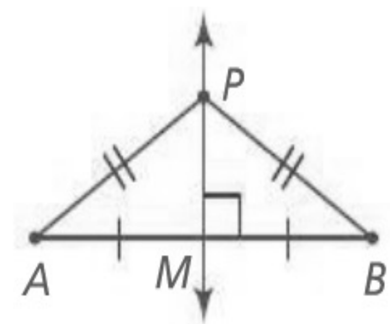
- Objective
  - I will be able to recognize a perpendicular bisector and an angle bisector. I will be able to use the associated theorem find missing angles, sides, and variables.
- Vocabulary

○ Equidistant	○ Bisector	○ Perpendicular Bisector Theorem
○ Angle Bisector Theorem		○ Converse of Perpendicular Bisector Theorem
○ Distance from a point to a line		

4.5 Bisectors

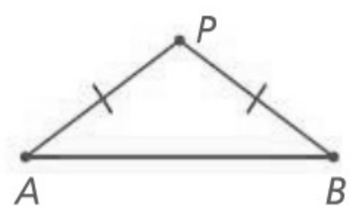
- Using the Perpendicular Bisector Theorem
  - *There is a special relationship between the points on the perpendicular bisector of a segment and the endpoints of segment.*
  - *Equidistant – the same distance*
  - *Perpendicular Bisector Theorem*
    - If a point is on the perpendicular bisector of a segment, it is equidistant from the endpoints of the segment.

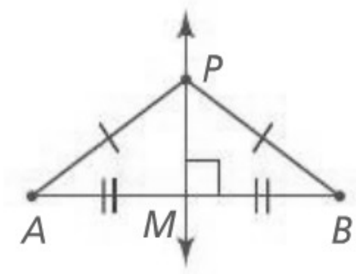




4.5 Bisectors

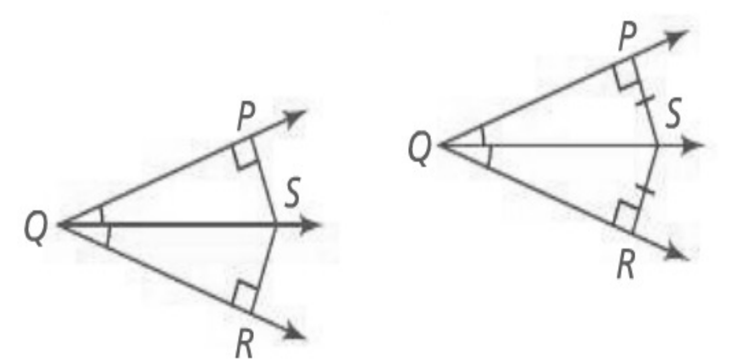
- *Converse of the Perpendicular Bisector Theorem*
  - If a point is equidistant from the endpoints of a segmen
    then it is on the perpendicular bisector of the segment.





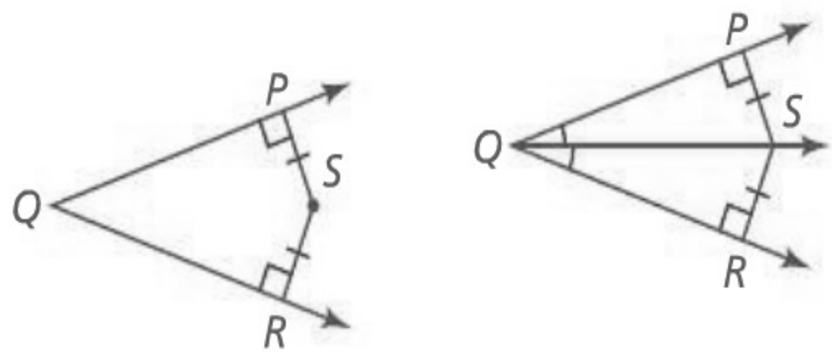
4.5 Bisectors

- Using the Angle Bisector Theorem
  - *The distance from a point to a line is the length of the perpendicular distance from the point to the line.*
  - *Shortest length from the line to the point.*
  - *Angle Bisector Theorem*
    - If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.



4.5 Bisectors

- *Converse of the Angle Bisector Theorem*
  - If a point in the interior of an angle is equidistant from t
    sides of the angle, then the point is on the angle bisecto

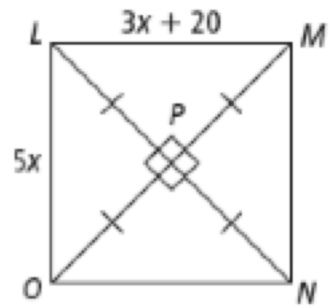


4.5 Bisectors - Practice

the figure at the right for Exercises 1–4.

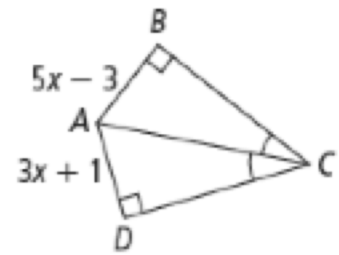
What is the relationship between $\overline{LN}$ and $\overline{MO}$?

Find LM.



Algebra Find the indicated values of the variables an

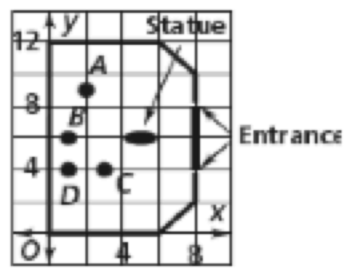
9. x, BA, DA



At the right is a layout for the lobby of a building placed on a coordinate grid.

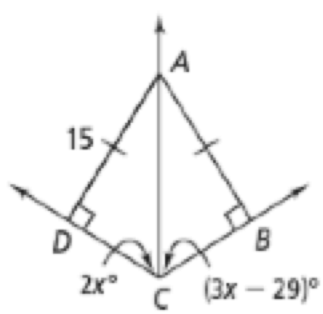
a. At which of the labeled points would a receptionist chair be equidistant from both entrances?

b. Is the statue equidistant from the entrances? How do you know?



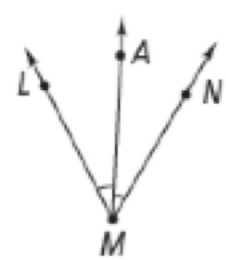
e the figure at the right for Exercises 6–8.

Find the value of x.



Writing Determine whether A must be on the bisector of ∠

11.



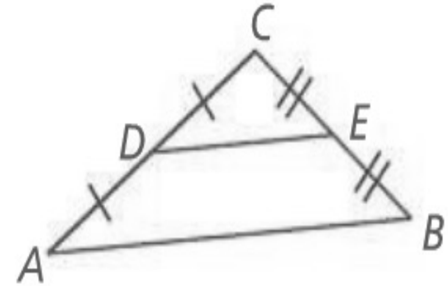
4.6 Midsegment & Side-Splitter Theorems

- Objective
  - I will be able to identify a midsegment, use the triangle midsegment theorem, the side-splitter theorem and its corollary, and the triangle-angle-bisector theorem to solve missing sides and variables.
- Vocabulary

○ Midsegment	○ Triangle Midsegment Theorem	○ Side-Splitter Theo
○ Triangle-Angle-Bisector Theorem	○ Corollary to Side-Splitter Theo	

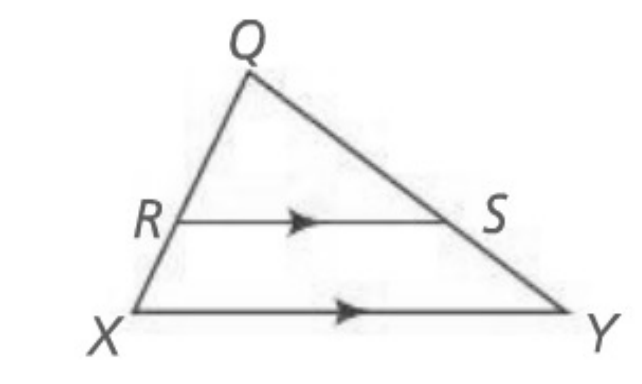
4.6 Midsegment & Side-Splitter Theorems

- Midsegment
  - *A midsegment of a triangle is a segment connecting the midpoints of two sides of a triangle.*
  - *Triangle Midsegment Theorem*
    - If a segment joins the midpoints of two sides of a triang then the segment is parallel to the third side and is half long.
    - Can be used to find the lengths of segments that might difficult to measure directly.



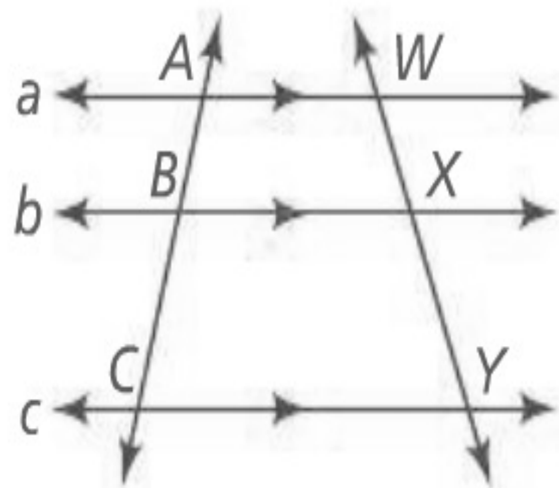
4.6 Midsegment & Side-Splitter Theorems

- Side-Splitter
  - *Side-Splitter Theorem*
    - If a line is parallel to one side of a triangle and intersect other two sides, then it divides those sides proportional
    - For the diagram below, *XR/RQ=YS/SQ*



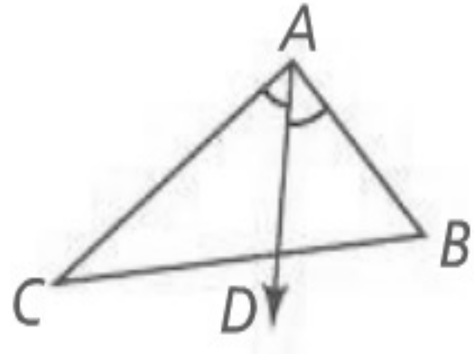
4.6 Midsegment & Side-Splitter Theorems

- *Corollary to the Side-Splitter Theorem*
  - If three parallel lines intersect two transversals, then the segments intercepted on transversals are proportional.
  - In the diagram below, *AB/BC=WX/XY*



4.6 Midsegment & Side-Splitter Theorems

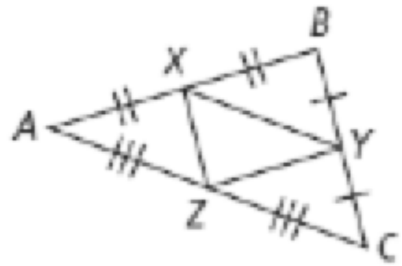
- *Triangle-Angle-Bisector Theorem*
  - If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to other two sides of the triangle.
  - In the diagram to the left, *CD/DB=CA/BA*



4.6 Midsegment & Side-Splitter Theorems Practice

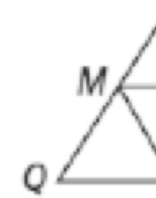
ame the triangle sides that are parallel to the given side.

. $\overline{AB}$



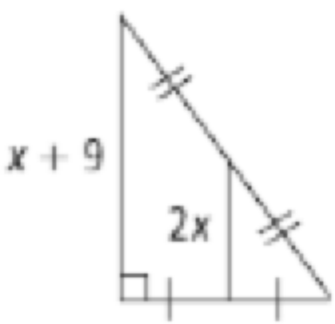
Points *M, N,* and *P* are the midpoints of the sides
$QR = 30$, $RS = 30$, and $SQ = 18$.

3. Find *MN*



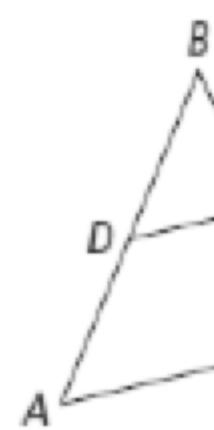
Algebra Find the value of *x*.

5.



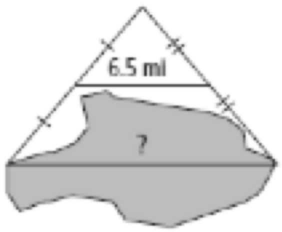
D is the midpoint of $\overline{AB}$. *E* is the midpoint of $\overline{CB}$.

7. If $m\angle A = 70$, find $m\angle BDE$.



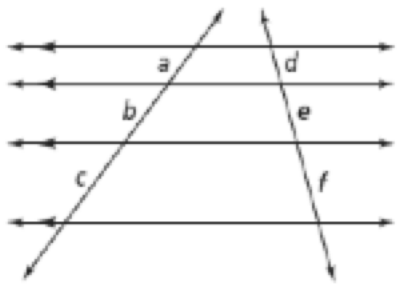
4.6 Midsegment & Side-Splitter Theorems Practice

d the distance across the lake in each diagram.



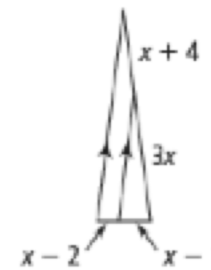
e the figure at the right to complete each proportion.

1. $\frac{a}{c} = \frac{\square}{f}$

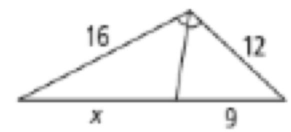


Algebra Solve for *x*.

13.



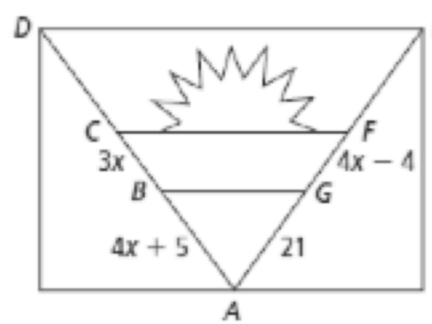
15.



17. The flag of Antigua and Barbuda is like the image at the right. In the image, $\overline{DE} \parallel \overline{CF} \parallel \overline{BG}$.

   a. An artist has made a sketch of the flag for a mural. The measures indicate the length of the lines in feet. What is the value of *x*?

   b. What type of triangle is ΔACF? Explain.



4.7 Similar Figures

- Objective
  - I will be able to identify similar figures and use the scale fa to find the original sizes. I will be able to dilate figures and dilations and scale factors to work out real-world problems

- Vocabulary

Similar Figures	○ Similar Polygons	○ Extended Proportions	
Scale Drawing	○ Scale	○ Dilation	○ Center of Dilation
Enlargement	○ Reduction	○ Scale Factor	

4.7 Similar Figures

- Similarity
  - Similar figures have the same shape but not necessarily th
    same size.
  - Symbol: ~
  - Two polygons are similar polygons if corresponding angles
    congruent and if lengths of corresponding sides are
    proportional.
  - Extended proportion: three or more equal proportions.

4.7 Similar Figures

- Scale
  - Scale factor, n: the ratio of corresponding linear measurem of two similar figures.
  - In a scale drawing, all lengths are proportional to their corresponding actual lengths.
    - Scale is the ratio that compares each length in a scale drawing to the actual length.
    - Scale can use different units (ex: 1cm = 50km)

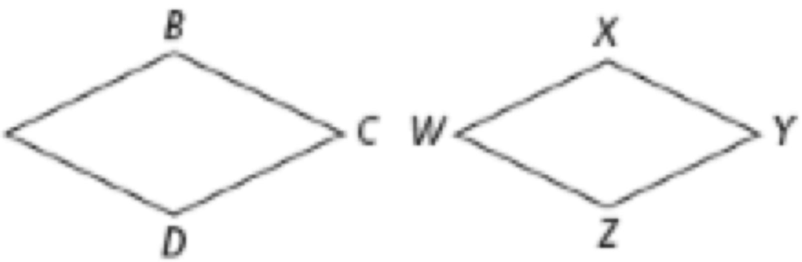
4.7 Similar Figures

- Dilations (Review)
  - *Produce similar figures.*
  - *Two types:*
    - Enlargement: makes a larger figure (n > 1)
    - Reduction: makes a smaller figure (0 < n < 1)
  - *Dilations and scale factors can help you understand real-w enlargements and reductions.*

4.7 Similar Figures - Practice

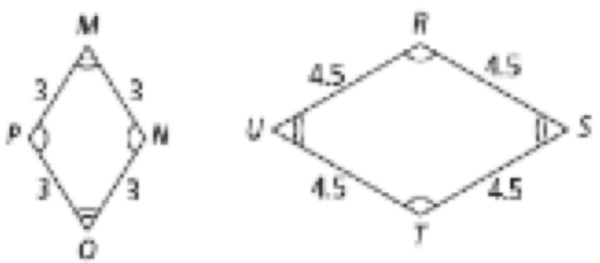
st the pairs of congruent angles and the extended proportion that relates the corresponding sides for e similar polygons.

. *ABCD ~ WXYZ*



etermine whether the polygons are similar. If so, write a similarity statement and give the scale factor. not, explain.

3.



. An architect is making a scale drawing of a building. She uses the scale 1 in. = 15 ft.

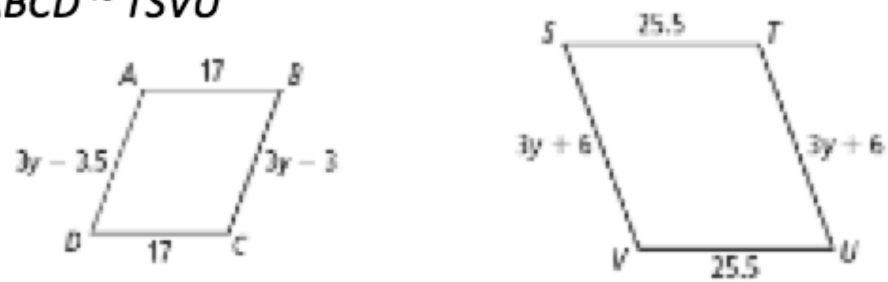
a. If the building is 48 ft tall, how tall should the scale drawing be?

b. If the building is 90 ft wide, how wide should the scale drawing be?

4.7 Similar Figures - Practice

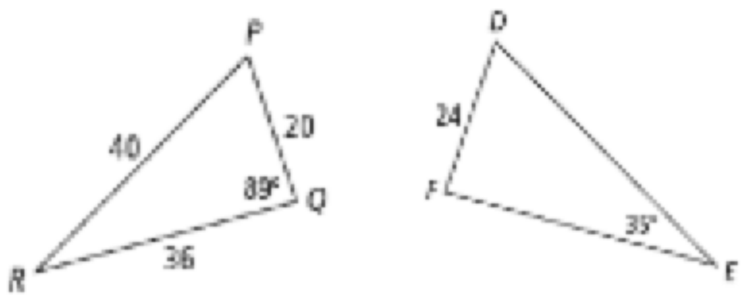
lgebra Find the value of *y*. Give the scale factor of the polygons.

. *ABCD* ~ *TSVU*



the diagram below, Δ *PRQ* ~ Δ *DEF*. Find each of the following.

1. *DE*



ou look at each object described in Exercises 13–14 under a magnifying glass. Find the actual imension of each object.

3. The image of a worm is 4 times its actual size and has a length of 7 cm.

4.8 Similarity

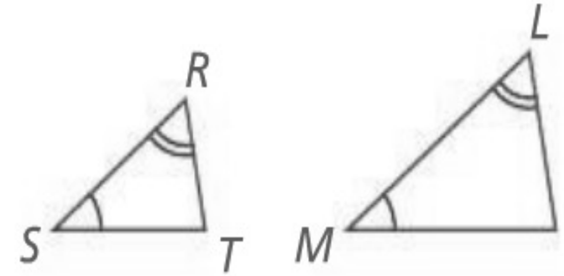
- Objective
  - I will be able to use postulates and theorems to identify si triangles. I will be able to use indirect measurement to fin actual lengths.

- Vocabulary

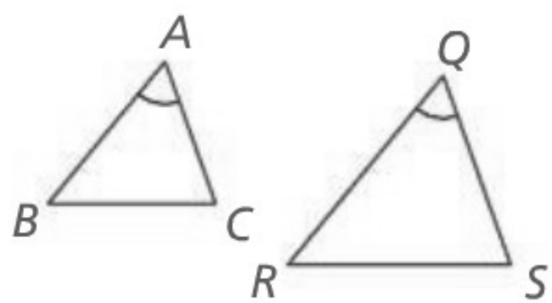
○ Indirect Measure	○ Angle-Angle Similarity Postulate
○ Side-Angle-Side Similarity Postulate	○ Side-Side-Side Similarity Postulate

4.8 Similarity

- Similarity Theorems/Postulates
  - *Angle-Angle Similarity (AA~) Postulate*
    - If two angles of one triangle are congruent to two angles another triangle, then the triangles are similar.



  - *Side-Angle-Side Similarity (SAS~) Theorem*
    - If an angle of one triangle is congruent to an angle of a second triangle, and the sides that include the two angl are proportional, then the triangles are similar.



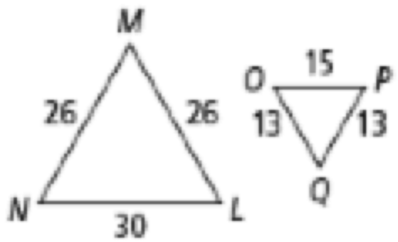
4.8 Similarity

- *Side-Side-Side Similarity (SSS~) Theorem*
  - If the corresponding sides of two triangles are proportio then the triangles are similar.
- Finding Lengths
  - *Similar triangles can be used to find lengths that cannot b measured easily.*
  - *Indirect measurement – a method of measurement that u formulas, similar figures and/or proportions.*

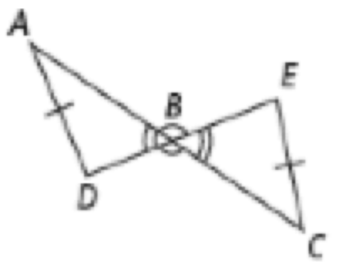
4.8 Similarity - Practice

Determine whether the triangles are similar. If so, write a similarity statement and name the postulate or theorem you used. If not, explain.

1.



3.

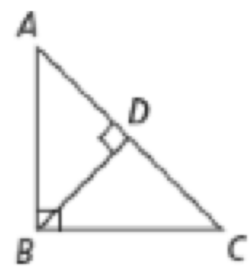


Explain why the triangles are similar. Then find the value of *x*.

5. $\overline{OP} \cong \overline{NP}, KN = 15,$
$LO = 20, JN = 9,$
$MO = 12$

Identify the similar triangles in each figure. Explain.

9.



7. A stick 2 m long is placed vertically at point *B*. The top of the stick is in line with the top of a tree as seen from point *A*, which is 3 m from the stick and 30 m from the tree. How tall is the tree?