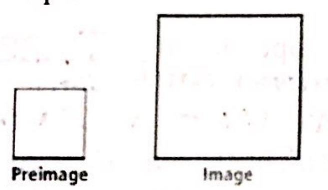
Name: ____________________

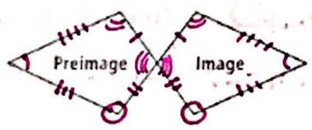
6.1 Translations (M1 8.1)

Obj.: I will be able to identify and write rules for geometric translations. I will be able to translate a shape in the coordinate plane.

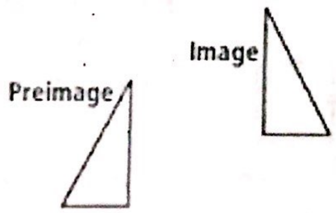
- Identifying a Rigid Motion
    - A transformation of a geometric figure is a function, or mapping, that results in a change in the position, shape, or size of the figure.
        - The original figure is the preimage.
        - The resulting figure is the image.
    - Certain transformations preserve distance and/or angles.
        - This means that the distance and/or angles do NOT change from the preimage to the image.
        - A transformation that preserves distance and angle measures is called a rigid motion.
        - Ex: Do the following transformations appear to be rigid motions? Explain.



No; angle measures are preserved, but distances have changed.

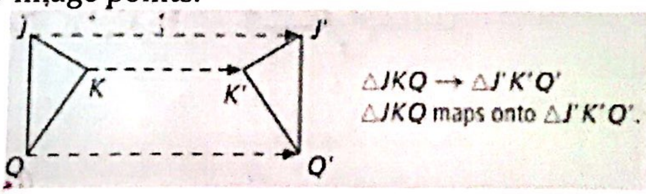


Yes; angle measures and distances have been preserved from preimage.



Yes; distances and angle measures are the same in the image as in the preimage.

- Naming Images and Corresponding Parts
    - A transformation maps every point of a figure onto its image and may be described with arrow notation (⟶).
    - Prime notation (′) is sometimes used to identify image points.



*Notice that corresponding parts are listed in the same order for both image & preimage. *

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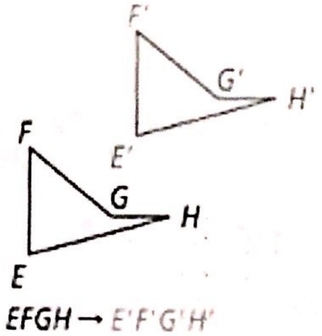
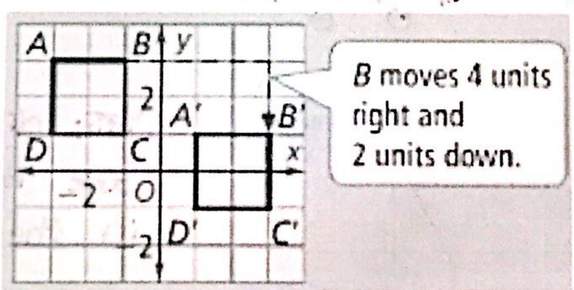
- Ex: In the diagram, EFGH → E'F'G'H', what are the images of $\angle F$ *and* $\angle H$? What are the pairs of corresponding sides?



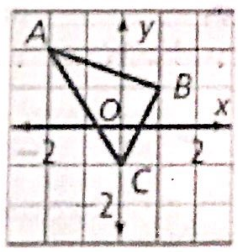
$\angle F'$ is the image of $\angle F$
$\angle H'$ is the image of $\angle H$

$\overline{EF} \to \overline{E'F'}$ $\overline{FG} \to \overline{F'G'}$
$\overline{EH} \to \overline{E'H'}$ $\overline{GH} \to \overline{G'H'}$

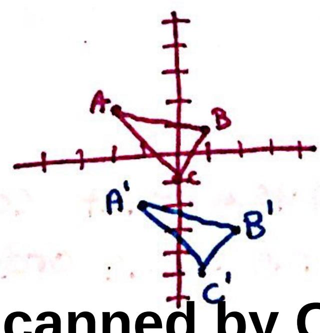
- Finding the Image of a Translation
  - A <u>translation</u> is a transformation that maps all points of a figure the dame distance in the same <u>direction</u>.
  - You write the translation that maps ΔABC onto $\Delta A'B'C'$ as <u>$T(\Delta ABC) = \Delta A'B'C'$</u>.
  - Translations have the following properties: If <u>$T(\Delta ABC) = \Delta A'B'C'$</u>,
    - AA' = BB' = CC' (all pts moved same dist.)
    - AB = A'B', BC = B'C', AC = A'C' (sides are same)
    - $m\angle A = m\angle A'$, $m\angle B = m\angle B'$, $m\angle C = m\angle C'$ (angle measures same)
  - For the translation of ABCD below, each point is translated 4 units right and 2 units down.
    - Therefore, each point <u>(x, y)</u> is mapped to <u>$(x+4, y-2)$</u>.
    - In function notation: $T_{<4,-2>}(ABCD) = A'B'C'D'$



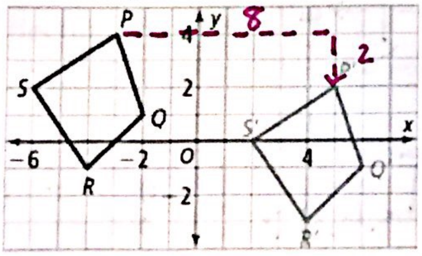
- Ex: What are the vertices of $T_{<1,-4>}(\Delta ABC)$? Copy and graph its image.



$$\begin{array}{l} A\ (-2, 2) \\ \quad\ \ (1, -4) \\ \hline A'\ (-1, -2) \end{array} \qquad \begin{array}{l} B\ (1, 1) \\ \quad\ (1, -4) \\ \hline B'\ (2, -3) \end{array} \qquad \begin{array}{l} C\ (0, -1) \\ \quad\ \ (1, -4) \\ \hline C'\ (1, -5) \end{array}$$



- Writing a Rule to Describe a Translation
    - You will need to know the coordinates of the vertices of both figures.
    - Find an algebraic relationship that maps each point of the preimage onto the image.
    - Use one pair of corresponding vertices to find the change in the horizontal direction x and the change in the vertical direction y. Then use other vertices to verify.
        - Ex: What is a rule that describes the translation that maps PQRS onto P'Q'R'S'?



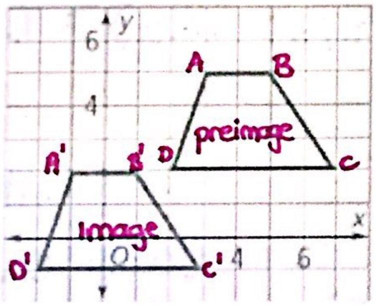
$P(-3,4)$ $\quad \Delta x: 5-(-3)=8$

$P'(5,2)$ $\quad \Delta y: 2-4=-2$

$(x,y) \rightarrow (x+8,\ y-2)$

$T_{<8,-2>}\ PQRS = P'Q'R'S'$

- The left figure is a translation of the right figure. Write a rule to describe the translation.



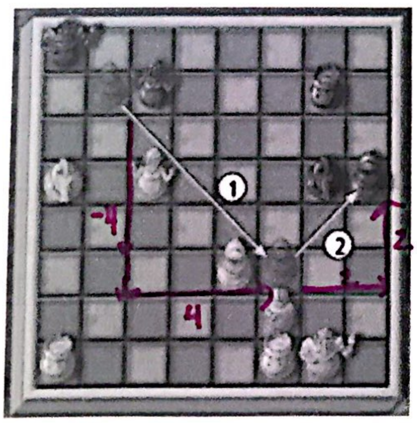
$A(3,5)$ $\quad \Delta x: -1-3=-4$

$A'(-1,2)$ $\quad \Delta y: 2-5=-3$

$(x,y) \rightarrow (x-4,\ y-3)$

$T_{<-4,-3>}\ ABCD = A'B'C'D'$

- Composing Translations
    - A composition of transformations is a combination of two or more transformations.
    - In general, the composition of any two translations is a translation.
        - Ex: The diagram at the right shows two moves of the black bishop in a chess game. Where is the bishop in relation to its original position?



Use (0,0) for original position.

$T_{<4,-4>}(x,y) = (x+4,\ y-4)$

$T_{<2,2>}(x,y) = (x+2,\ y+2)$

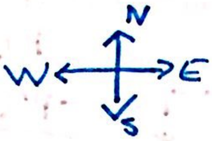
$T_{<4,-4>}(0,0) = (0+4,\ 0-4) = (4,-4)$

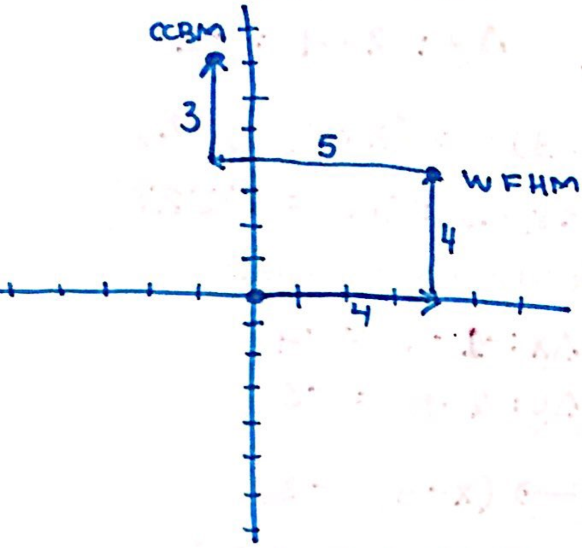
$T_{<2,2>}(4,-4) = (4+2,\ -4+2) = \boxed{(6,-2)}$

$$\begin{array}{r} 4,-4 \\ \underline{2,2} \\ 6,-2 \end{array}$$

$\boxed{T_{<6,-2>}(x,y) = (x+6,\ y-2)}$

Ex: You are visiting San Fransisco. From your hotel near Union Square, you walk 4 blocks east and 4 blocks north to the Wells Fargo History Museum. Then you walk 5 blocks west and 3 blocks north to the Cable Car Barn Museum. Where is the Cable Car Barn Museum in relation to your hotel.





$$T_{\langle 4,4\rangle}(x,y) = (x+4,\ y+4)$$

$$T_{\langle -5,3\rangle}(x,y) = (x-5,\ y+3)$$

$$\begin{array}{r} (4,4) \\ (-5,3) \\ \hline (-1,7) \end{array}$$

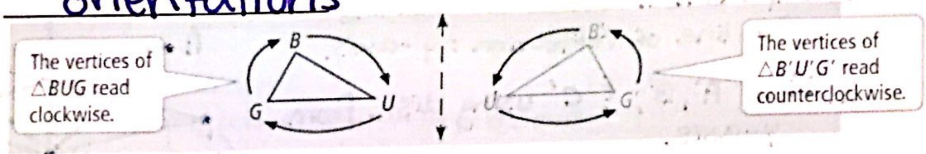
From the hotel, the Cable car Barn Museum is 1 block West and 7 blocks north.

Name: ____________________

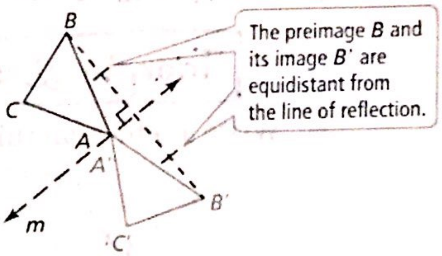
6.2 Reflection (M1 8.2)

Obj.: I will be able to identify and write rules for reflections. I will be able to reflect a shape across a given line.

- Reflecting a Point Across a Line
  - When you reflect a figure, the shapes have opposite orientations



  - Reflecting over a line:
    - Each point is the same distance from the line in the image as in the pre-image.
    - In order to precisely define reflections, you need to use the perpendicular bisector of a segment. } line ⊥ @ midpt.
  - flip! A reflection across line m, called the line of reflection is a transformation with the following properties:
    - If point A is on line m, then the image of A is itself (A' = A)
    - If point B is not on line m, then m is the perpendicular bisector of BB'.
  - You write the reflection across m that takes P to P' as $R_m(P) = P'$.



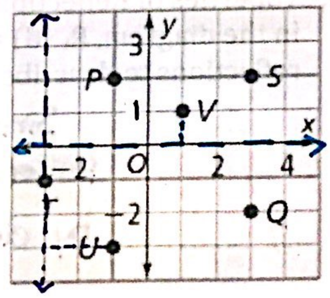
  - You can use the equation of the line when writing a reflection in function notation (Ex: $R_{y=x}$ describes a reflection over the line y = x)
    - Ex: Find the coordinates of each image.
      - $R_{x=-3}$ (U)

        U is 2 units right of x = -3, so U' will be 2 units left

        $\boxed{U'(-5,-3)}$
      - $R_{x\text{-axis}}$ (V)

        V is 1 unit above x-axis, so V' will be 1 unit below

        $\boxed{V'(1,-1)}$

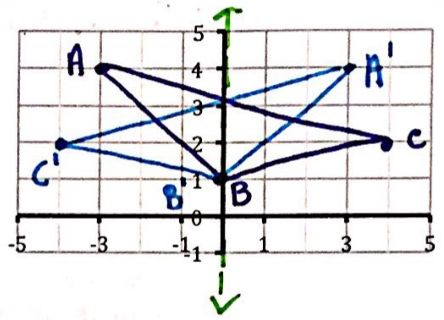


- Graphing a Reflection Image
  - Properties of reflections:
    - Reflections preserve distance and angle measures. → rigid motions!
      - If $R_m(A) = A'$ and if $R_m(B) = B'$, then AB = A'B'

Name: ______________________

- If $R_m(\angle ABC) = \angle A'B'C'$, then $m\angle ABC = m\angle A'B'C'$
  - Reflections map each point of the preimage to one and only one corresponding point of its image.
    - $R_m(A) = A'$ if and only if $R_m(A') = A$
- Ex: Graph points A(−3, 4), B (0,1), and C(4, 2). Graph and label $R_{y\text{-axis}}(\Delta ABC)$.

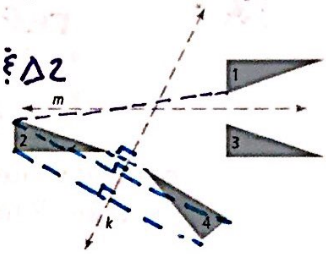
1. Graph ΔABC. Show dashed line for line of reflection: y-axis.
2. Find A', B', & C' using dist. from y-axis.
3. Graph $\Delta A'B'C'$



- Writing a Reflection Rule
  - Each triangle in the diagram is a reflection of another triangle across one of the given lines. How can you describe triangle 2 by using a reflection rule?

X $\Delta 2$ & $\Delta 3$ have same orientation.
X neither k nor m are $\perp$ to the line between $\Delta 1$ & $\Delta 2$
✓ line k is $\perp$ to all lines btwn $\Delta 2$ & $\Delta 4$

$\boxed{\text{Triangle } 2 = R_k(\text{Triangle } 4)}$



  - Write a reflection rule to describe Figure 4. Justify your answer.

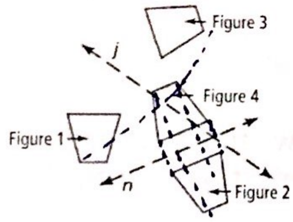
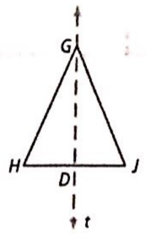


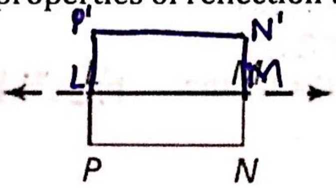
Figure 4 = R_n(Figure 2)
b/c line n is $\perp$ bisector of line segments between corresp. vertices of Figures 4 & 2.

- Using Properties of Reflections
  - In the diagram, $R_t(G) = G$, $R_t(H) = J$, and $R_t(D) = D$. Use the properties of reflections to describe how you know that ΔGHJ is an isosceles triangle.



Since $R_t(G) = G$, $R_t(H) = J$ and reflections preserve distance, $R_t(\overline{GH}) = \overline{GJ}$. So, GH = GJ and, by definition, ΔGHJ is an isosceles Δ.

  - Sketch $R_{\overline{LM}}(LMNP)$. What figure results from the reflection? Use the properties of reflection to justify your answer. Note: LM = 2MN.



square. Since $R_{\overline{LM}}(M) = M$, $R_{\overline{LM}}(N) = N'$, and reflections preserve distance, $R_{\overline{LM}}(\overline{MN}) = \overline{MN'}$, so MN = MN' and NN' = 2MN. Since LM = 2MN and NN' = 2MN, by substitution LM = NN'. Therefore, in the new figure PNN'P' the length = width, so the figure is a square.

Name: ______________________

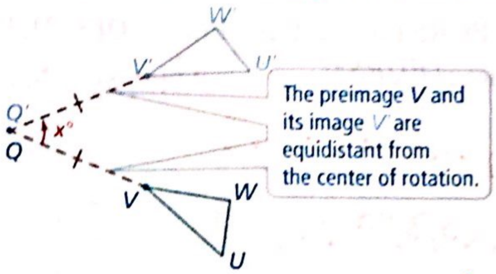
6.3 Rotations (M1 8.3)

Obj.: I will be able to identify and write rules for rotations. I will be able to rotate a shape about a given point.

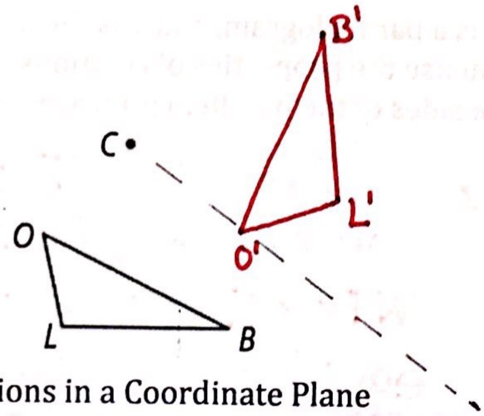
Turn!

RIGID MOTIONS!

- Drawing a Rotation Image
  - A rotation of $x°$ about a point Q, called the center of rotation is a transformation with two properties:
    - The image of Q is itself (Q' = Q)
    - For any other point V, QV' = QV and $m\angle VQV' = x$.

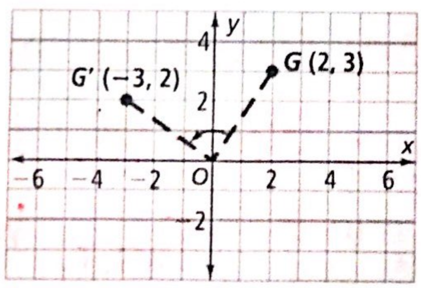


  - The number of degrees a figure rotates is the angle of rotation.
  - The symbol for a rotation of x° of a shape, like ΔUVW, is $r_{(x°, Q)}(\Delta UVW) = \Delta U'V'W'$ (about point Q)
    - Ex: What is the image of $r_{100°, C}(\Delta LOB)$?

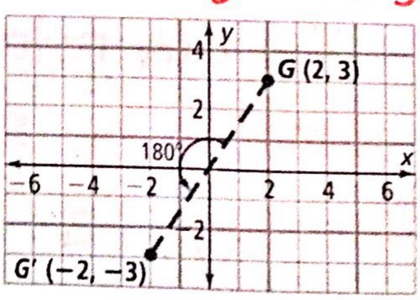


- Drawing Rotations in a Coordinate Plane
  - When a figure is rotated 90°, 180°, 270°, or 360° about the origin in a coordinate plane, you can use the following rules:

$r_{(90°, O)}(x, y) = (-y, x)$

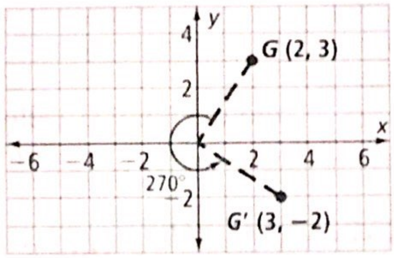


$r_{(180°, O)}(x, y) = (-x, -y)$

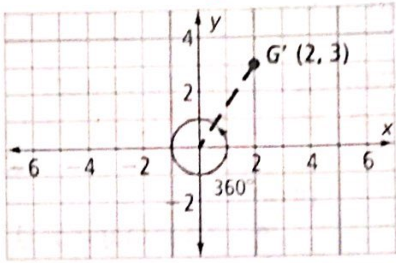


Name: ____________________

$r_{(270°,O)}(x,y) = (y,-x)$



$r_{(360°,O)}(x,y) = (x,y)$



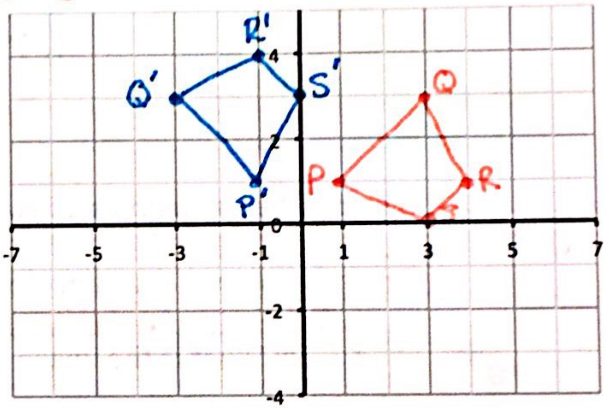
- Ex: PQRS has vertices P(1,1), Q(3, 3), R (4, 1), and S (3, 0). What is the graph of $r_{90°,O}(PQRS)$? $r_{(90°,O)}(x,y) = (-y,x)$

$P' = r_{(90°,O)}(1,1) = (-1,1)$

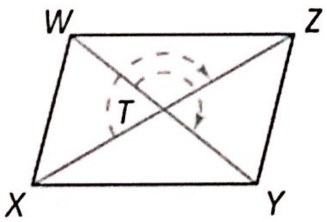
$Q' = r_{(90°,O)}(3,3) = (-3,3)$

$R = r_{(90°,O)}(4,1) = (-1,4)$

$S = r_{(90°,O)}(3,0) = (0,3)$



- Using Properties of Rotations
  - In the diagram, WXYZ is a parallelogram, and T is the midpoint of the diagonals. How can you use the properties of rotations to show that the lengths of the opposite sides of the parallelogram are equal?



Because T is the midpoint of the diagonals, XT = ZT and WT = YT. Since W and Y are equidistant from T, and the measure of ∠WTY = 180, you know that $r_{(180,T)}(W) = Y$. Similarly, $r_{(180,T)}(X) = Z$. You can rotate every point on $\overline{WX}$ in the same way, so $r_{(180,T)}\overline{WX} = \overline{YZ}$. Likewise, you can map $\overline{WZ}$ to $\overline{YX}$ with $r_{(180,T)}\overline{WZ} = \overline{YX}$. Because rotations are rigid motions and preserve distance, WX = YZ and WZ = YX.

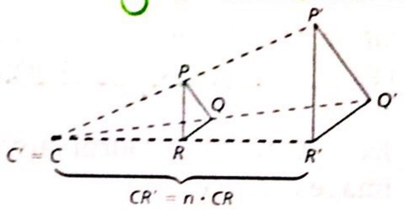
Name: ______________________

6.4 Dilations (M2 6.6)

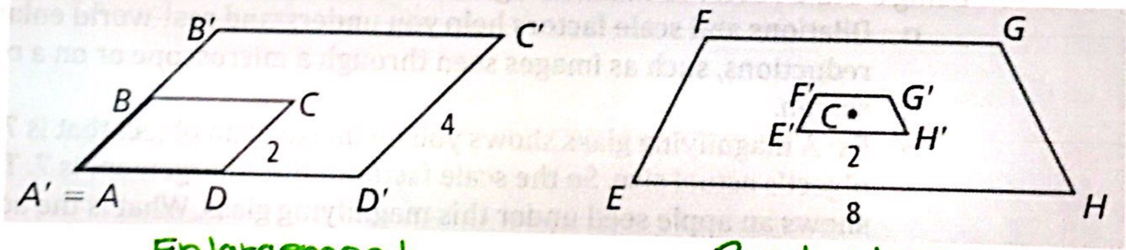
Obj.: I will be able to identify and write rules for dilations. I will be able to dilate a shape about a given point.

- Finding a Scale Factor
  - A dilation with center of dilation C and scale factor $n, n > 0$, can be written as $D_{(n,C)}$.
  - Dilations are transformations with the following properties:
    - The image of C is itself ($C' = C$)
    - For any other point R, R' is on $\overrightarrow{CR}$ and $CR' = n * CR$ or $n = \frac{CR'}{CR}$
    - Dilations preserve angle measure.

NOT RIGID → MOTIONS



  - Scale factor n of a dilation is the ratio of a length of the image to the corresponding length in the preimage, with the length always in the numerator.
  - Two types of dilations:
    - enlargement if $n > 1$
    - reduction if $0 < n < 1$



Enlargement
center A, $n = 2$

Reduction
center C, $n = \frac{1}{4}$

  - Ex: Is $D_{(n,X)}(\Delta XTR) = \Delta X'T'R'$ an enlargement or a reduction? What is the scale factor n of the dilation?

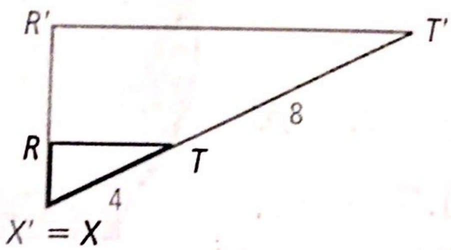


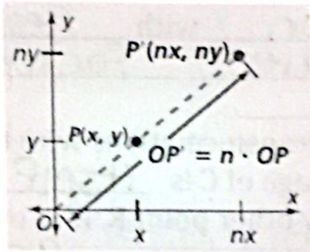
Image larger than preimage
→ $\boxed{\text{enlargement}}$

$$n = \frac{X'T'}{XT} = \frac{4+8}{4} = \frac{12}{4} = \boxed{3}$$

$$\boxed{D_{(3,X)}(\Delta XTR) = \Delta X'T'R'}$$

Name: ______________________

- Finding a Dilation Image
    - You can find the coordinates of a dilation in the coordinate plane with the origin as the center of dilation by multiplying the preimage coordinates by the scale factor.

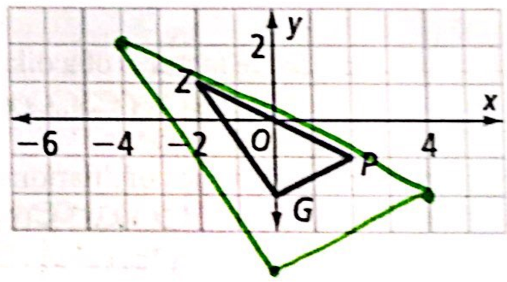


    - The dilation of scale factor n with center of dilation at the origin can be written as $D_n(x,y)=(nx,ny)$.
        - Ex: What are the coordinates of the vertices of $D_2(\Delta PZG)$? Graph the image of ΔPZG.

$D_2(P)=(2\cdot 2,\ 2\cdot -1)=(4,-2)$

$D_2(Z)=(2\cdot -2,\ 2\cdot 1)=(-4,2)$

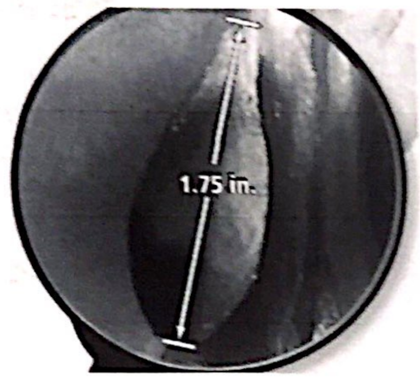
$D_2(G)=(2\cdot 0,\ 2\cdot -2)=(0,-4)$



- Using a Scale Factor to Find a Length
    - Dilations and scale factors help you understand real-world enlargements and reductions, such as images seen through a microscope or on a computer screen.
    - Ex: A magnifying glass shows you an image of an object that is 7 times the object's actual size. So the scale factor of the enlargement is 7. The photo shows an apple seed under this magnifying glass. What is the actual length of the apple seed?

$1.75\text{cm} = 7\cdot p$

$0.25\text{cm} = p$



The length of the original apple seed is 0.25cm.