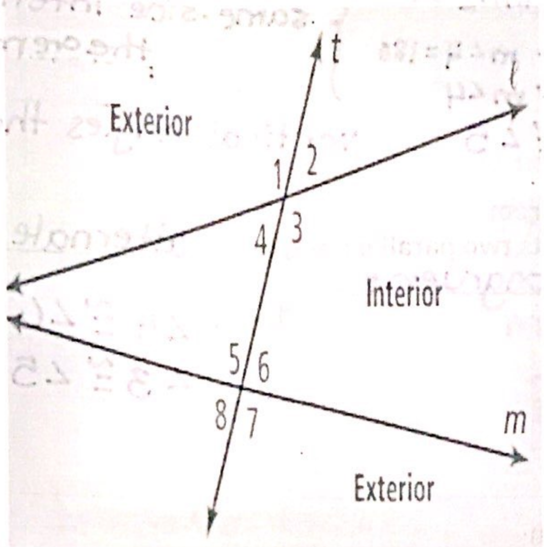
Unit 7 – Congruence Name: ____________________

7.1 Special Angle Relationships (M2 2.2)

Obj.: Using lines cut by a transversal, I will be able to identify angle pairs. When the lines cutting the transversal are parallel, I will be able to identify supplementary and congruent angles. I will also be able to use angle relationships to solve for x algebraically.

- Identifying Angle Pairs

other diagrams may be # differently



When the lines are parallel:

(side-by-side)
Linear Pairs: ∠1 & ∠2, ∠4 & ∠3, ∠5 & ∠6, ∠8 & ∠7
∠1 & ∠4, ∠2 & ∠3, ∠5 & ∠8, ∠6 & ∠7

Alternate Exterior Angles: ∠1 & ∠7, ∠2 & ∠8

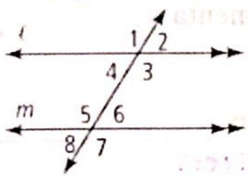
Alternate Interior Angles: ∠4 & ∠6, ∠3 & ∠5

Same Side Interior Angles: ∠4 & ∠5, ∠3 & ∠6

Corresponding Angles: ∠1 & ∠5, ∠2 & ∠6, ∠4 & ∠8, ∠3 & ∠7

Vertical Angles: ∠1 & ∠3, ∠2 & ∠4, ∠5 & ∠7, ∠6 & ∠8
(diagonal)

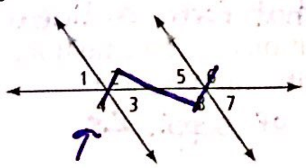
- Identifying Supplementary Angles
  - Recall that vertical angles are angles formed when two lines intersect (opposite angles). } congruent
  - Special angle pairs formed by parallel lines and a transversal are (m∠ =) → congruent, supplementary (+ to 180), or both ← (90°).
  - Same-Side Interior Angles Postulate
    - If a transversal intersects two parallel lines, then same-side interior angles are supplementary.
    - If… $l \parallel m$ Then… $m\angle 4 + m\angle 5 = 180$
      $m\angle 3 + m\angle 6 = 180$



  - Ex: The measure of ∠3 is 55. Which angles are supplementary to ∠3? How do you know?

$180 - 55 = 125°$

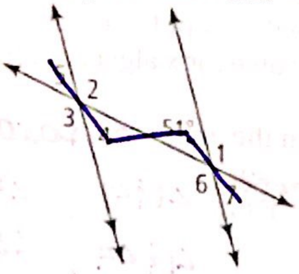
∠2, ∠4, ∠6, ∠8 each = 125°



Zig-zag (or Z) are congruent to each other

$m\angle 3 + m\angle 2 = 180$ linear pairs are suppl.
$\angle 2 \cong \angle 4$ vertical angles theorem
$m\angle 3 + m\angle 8 = 180$ same-side interior angles theorem
$\angle 8 \cong \angle 6$ vertical angles theorem.

- Ex: Identify all the numbered angles that are congruent to the given angle. Justify your answers.



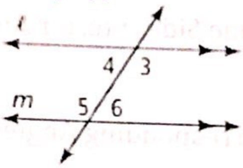
$\angle 7, \angle 4, \angle 5 \cong 51°$

$51° \cong \angle 7$ vertical angles theorem

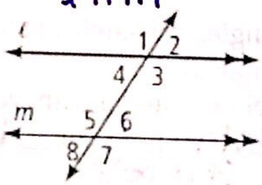
$51° + m\angle 2 = 180$
$m\angle 2 + m\angle 4 = 180$
$51° \cong m\angle 4$
} same-side interior angles postulate

$\angle 4 \cong \angle 5$ vertical angles theorem

- Proving an Angle Relationship
  - Alternate Interior Angles Theorem
    - If a transversal intersects two parallel lines, then alternate interior angles are congruent.
    - If... $l \parallel m$ Then... $\angle 4 \cong \angle 6$, $\angle 3 \cong \angle 5$



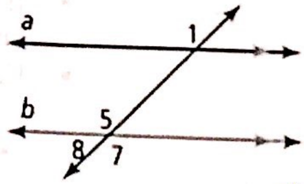
  - Corresponding Angles Theorem
    - If a transversal intersects two parallel lines, then corresponding angles are congruent.
    - If... $l \parallel m$ Then... $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, $\angle 4 \cong \angle 8$



2 column Proof

cannot jump from given directly to "prove"

- Ex: Proving an Angle Relationship
  Given: a || b
  Prove: ∠1 and ∠8 are supplementary.



always start w/ given

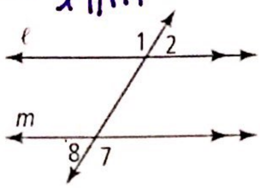
Statement	Reason
1. a||b	1. Given
2. $\angle 1 \cong \angle 5$	2. If lines are ||, then corresponding ∠s are ≅
3. $m\angle 1 = m\angle 5$	3. Congruent ∠s have equal measures.
4. ∠5 & ∠8 are supplementary	4. ∠s that form a linear pair are suppl.
5. $m\angle 5 + m\angle 8 = 180°$	5. Definition of supplementary ∠s
6. $m\angle 1 + m\angle 8 = 180°$	6. Substitution Property
7. ∠1 and ∠8 are supplementary	7. Def. of suppl. ∠s.

always end w/ "prove" statement

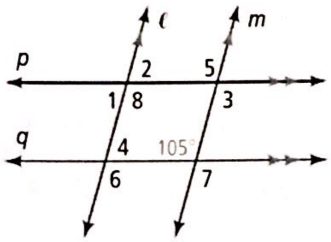
min 3 steps

Unit 7 – Congruence Name: ____________________

- Finding Measures of Angles
  - Alternate Exterior Angles Theorem
    - If a transversal intersects two parallel lines, then alternate exterior angles are ___congruent___.
    - If… $\ell \parallel m$ Then… $\angle 1 \cong \angle 7$, $\angle 2 \cong \angle 8$



  - Ex: What are the measures of ∠3 and ∠4? Which theorem or postulate justifies each answer?

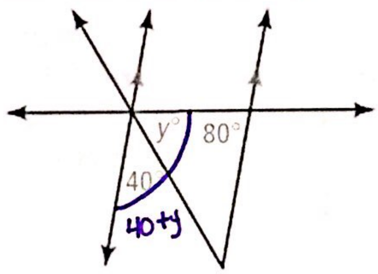


$\boxed{m\angle 3 = 105^\circ}$ alternate interior angles thm

$m\angle 4 + 105 = 180$ same-side interior angles postulate

$\boxed{m\angle 4 = 75^\circ}$

- Finding an Angle Measure
  - You can combine theorems and postulates with your knowledge of ___algebra___ to find angle measures.
    - Ex: What is the value of y?

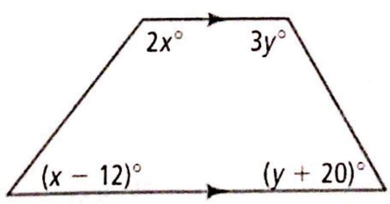


$(40+y)+80=180$

$y+120=180$

$\boxed{y=60^\circ}$

    - Ex: In the figure below, what are the values of x and y?



$3y + (y+20) = 180$

$4y + 20 = 180$

$4y = 160$

$\boxed{y=40}$

$2x + (x-12) = 180$

$3x - 12 = 180$

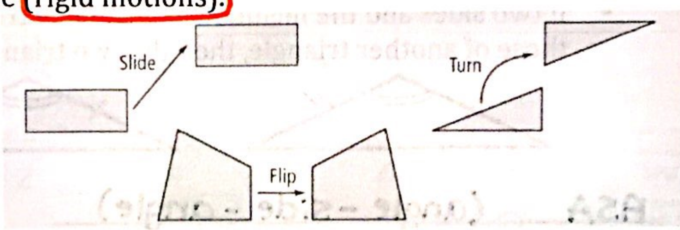
$3x = 192$

$\boxed{x=64}$

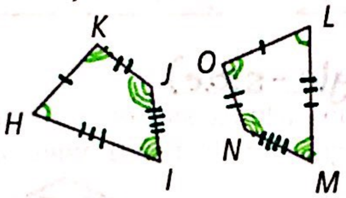
7.3 Definition of Congruence Figures and Triangle Congruence (M2 3.1-3,6)

Obj.: I will be able to identify corresponding parts of congruent triangles and use them to algebraically solve problems. I will also be able to identify whether two triangles are congruent by any of the five triangle congruence theorems.

- Finding Congruent Parts
  - Recall that congruent figures have the same size and shape. When two figures are congruent, you can slide, flip, or turn one so that it fits exactly on the other one (rigid motions).



  - Congruent polygons have congruent corresponding parts.
  - The order of the letters in a congruence statement matters!
    - Ex: HIJK ≅ LMNO



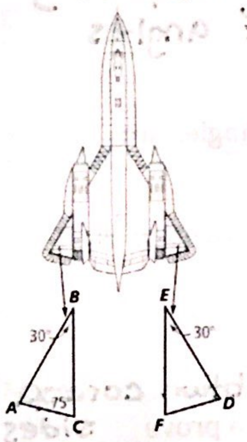
$\angle H \cong \angle L$ $\quad \angle I \cong \angle M$

$\angle J \cong \angle N$ $\quad \angle K \cong \angle O$

$\overline{HK} \cong \overline{LO}$ $\quad \overline{JI} \cong \overline{NM}$

$\overline{KJ} \cong \overline{ON}$ $\quad \overline{IH} \cong \overline{ML}$

- Using Congruent Parts
  - The wings of an SR-71 Blackbird aircraft suggest congruent angles. What is $m\angle D$?



$30 + 75 + m\angle A = 180$

$105 + m\angle A = 180$

$m\angle A = 75°$

$\angle A \cong \angle D$

$m\angle A = m\angle D$

$\boxed{75° = m\angle D}$

  - If $\Delta ABC \cong \Delta DEF$, identify congruent angles. If $\angle A = x + 5$ and $\angle D = 2x$, solve for x.

$\angle A \cong \angle D$

$x + 5 = 2x$

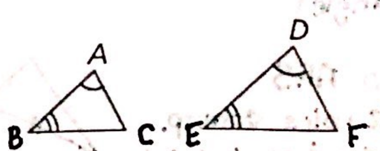
$-x \qquad -x$

$\boxed{5 = x}$

$m\angle A = 5 + 5 = \boxed{10}$

$m\angle D = 2(5) = \boxed{10}$

- Proving Triangles Congruent
  - Third Angles Theorem
    - If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.
    - If... $\angle A \cong \angle D$ & $\angle B \cong \angle E$ Then... $\angle C \cong \angle F$



Unit 7 – Congruence Name: ____________________

- Five Triangle Congruence Theorems/Postulates
  - SSS (side - side - side)
    - If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent

3 corresp. ≅ sides

  - SAS (side - angle - side)
    - If two sides and the included angle of one triangle are congruent to those of another triangle, then the two triangles are congruent.

angle between corresponding ≅ sides

  - ASA (angle - side - angle)
    - If two angles and the included side of one triangle are congruent to those of another triangle, then the two triangles are congruent.

side between corresponding congruent angles

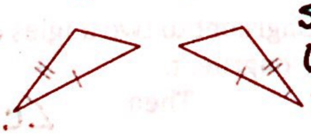
  - AAS (angle - angle - side)
    - If two angles and a non-included side of one triangle are congruent to those of another triangle, then the two triangles are congruent.

side not btwn corresponding ≅ angles

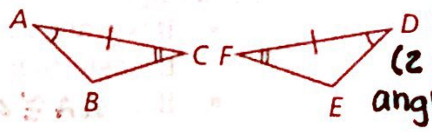
  - HL (hypotenuse - leg)
    - If the hypotenuse and one leg of a right triangle are congruent to those of another right triangle, then the two triangles are congruent.

RIGHT ∠ not btwn corresp. ≅ sides

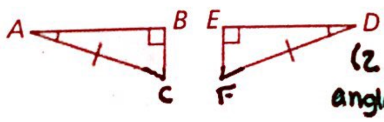
  - Examples: Which of the above theorems/postulates would you use to prove the triangles congruent? If there is not enough information to prove the triangles congruent, write *not enough information*. Explain your answer.



SAS (2 pairs corresp. sides & included angle)



ASA (2 pairs corresp. angles & included side)



AAS (2 pairs corresp. angles & non-included side)

SSS (3 pairs corresp. sides)

Unit 7 – Congruence Name: ____________________

7.4 Triangle Congruence (M2 3.1-3,6)

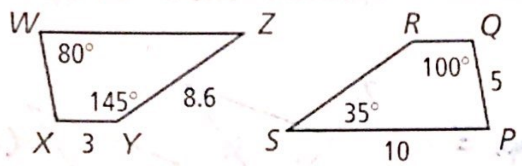
Obj.: I will be able to identify corresponding parts of congruent triangles and use them to algebraically solve problems. I will also be able to identify whether two triangles are congruent by any of the five triangle congruence theorems.

Exercises

RSTUV ≅ *KLMNO*. Complete the congruence statements. (order of letters !!)

1. $\overline{TS} \cong$ __?__ $\overline{ML}$
2. $\angle N \cong$ __?__ $\angle U$
3. $\overline{LM} \cong$ __?__ $\overline{ST}$
4. $VUTSR \cong$ __?__ ONMLK

WXYZ ≅ *PQRS*. Find each measure or length.



$\angle W \cong \angle P$ $\overline{WX} \cong \overline{PQ}$
$\angle X \cong \angle Q$ $\overline{WZ} \cong \overline{PS}$
$\angle Y \cong \angle R$ $\overline{XY} \cong \overline{QR}$
$\angle Z \cong \angle S$ $\overline{YZ} \cong \overline{RS}$

5. $m\angle P$ 80°
6. QR 3
7. WX 5
8. $m\angle Z$ 35°
9. $m\angle X$ 100°
10. $m\angle R$ 145°

Exercises

11. In $\triangle HFD$, what angle is included between $\overline{DH}$ and $\overline{DF}$? $\angle D$

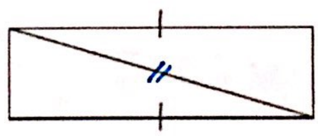
12. In $\triangle OMR$, what side is included between $\angle M$ and $\angle R$? $\overline{MR}$

13. Not enough info
14. Not enough info

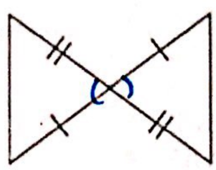
only have 2 corresp. parts; need 3

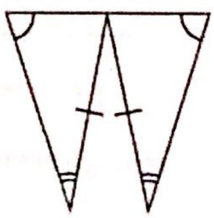
15. SAS (use vertical angles)
16. AAS (side not btwn angles)

Which postulate or theorem, if any, could you use to prove the two triangles congruent? If there is not enough information to prove the triangles congruent, write *not enough information.*

13. 

14. 

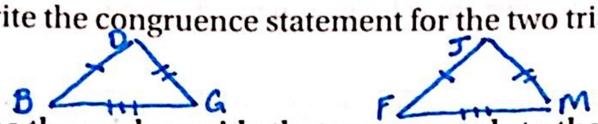
15. 

16. 

Name: ______________

Do you know HOW?

1. Two triangles have the following pairs of congruent sides: $\overline{BD} \cong \overline{FJ}$, $\overline{DG} \cong \overline{JM}$, and $\overline{GB} \cong \overline{MF}$. Write the congruence statement for the two triangles.



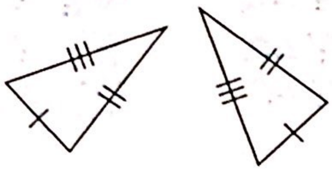
$\boxed{\triangle BDG \cong \triangle FJM}$

$\triangle QRS \cong \triangle TUV$. Name the angle or side that corresponds to the given part.

2. $\angle Q \cong \angle T$ 3. $\overline{RS} \cong \overline{UV}$ 4. $\angle S \cong \angle V$ 5. $\overline{QS} \cong \overline{TV}$

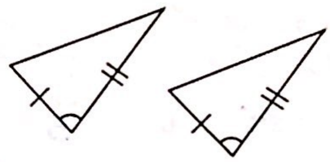
State the postulate or theorem that can be used to prove the triangles congruent. If you cannot prove the triangles congruent, write *not enough information.*

6.



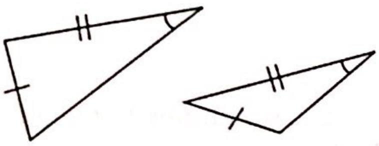
SSS
(3 corresp. ≅ sides)

7.



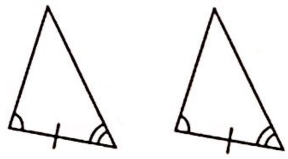
SAS
(2 corresp. ≅ sides w/ included ∠)

8.



not enough information
(no SSA)

9.



ASA
(2 corresp. ≅ ∠s w/ included side)

Unit 7 – Congruence   Name: ______________________________

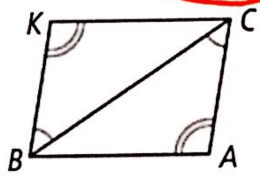
7.5 Corresponding Parts of Congruent Triangles are Congruent (M2 3.4)

Obj.: I will be able to prove parts of congruent triangles to be congruent.

- Proving Parts of Triangles Congruent
  - With SSS, SAS, ASA, and AAS, you know how to use three congruent parts of two triangles to show that the triangles are congruent.
  - If you know two triangles are congruent, then you know that every pair of their corresponding parts is also congruent.
    - Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
    - Ex: (Flow Proof)

      Given: $\angle KBC \cong \angle ACB, \angle K \cong \angle A$

      Prove: $\overline{KB} \cong \overline{AC}$



$\angle KBC \cong \angle ACB$ (Given) → $\overline{BC} \cong \overline{CB}$ (Reflexive property of ≅)

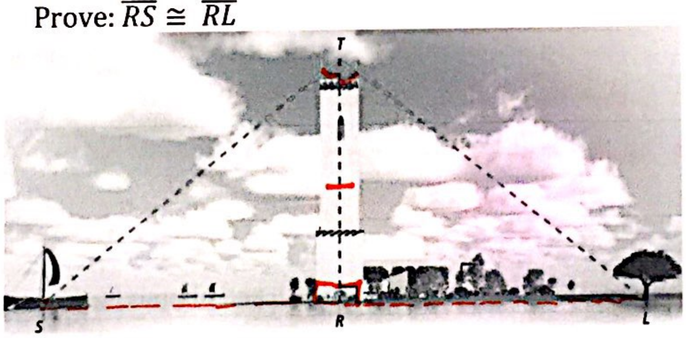
$\angle K \cong \angle A$ (Given)

→ $\triangle KBC \cong \triangle ACB$ (AAS Thm) → $\overline{KB} \cong \overline{AC}$ (CPCTC)

- Using Corresponding Parts to Measure Distance
  - Thales, a Greek philosopher, is said to have developed a method to measure the distance to a ship at sea. He made a compass by nailing two sticks together. Standing on top of a tower, he would hold one stick vertical and tilt the other until he could see the ship S along the line of the tilted stick. With this compass setting, he would find a landmark L on the shore along the line of the tilted stick. How far would the ship be from the base of the tower?

    Given: $\angle TRS$ *and* $\angle TRL$ *are right angles*, $\angle RTS \cong \angle RTL$

    Prove: $\overline{RS} \cong \overline{RL}$



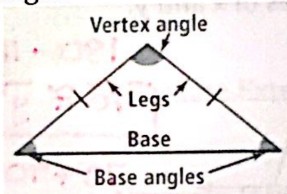
Statements	Reasons
1. $\angle RTS \cong \angle RTL$	1. Given
2. $\overline{TR} \cong \overline{TR}$	2. Reflexive Property of ≅
3. $\angle TRS$ & $\angle TRL$ are right ∠s	3. Given
4. $\angle TRS \cong \angle TRL$	4. All right angles are congruent
5. $\triangle TRS \cong \triangle TRL$	5. ASA Postulate
6. $\overline{RS} \cong \overline{RL}$	6. CPCTC

<u>**Unit 7 – Congruence**</u> Name: ______________________

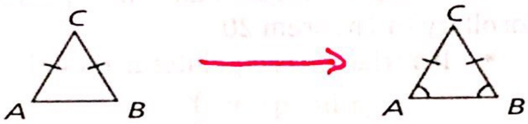
7.6 Isosceles Triangles (M2 3.5)

Obj.: I will be able to identify isosceles and equilateral triangles. I will be able to use the Isosceles Triangle Theorem, its converse, and their corollaries to calculate angle and side measures.

- Using the Isosceles Triangle Theorems
  - Recall: An isosceles triangle has two congruent legs.
  - Anatomy of an isosceles triangle:

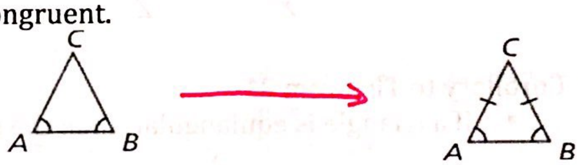


  - Isosceles Triangle Theorem (Thm 22)
    - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

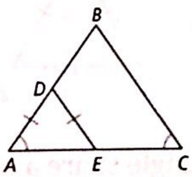


  - Converse of the Isosceles Triangle Theorem (Thm 21)
    - If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

means to reverse "if" and "then" statements

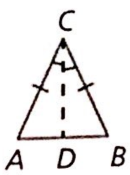


  - Ex: Is $\overline{AB} \cong \overline{CB}$? Explain.



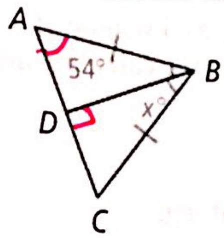
Yes. Since $\angle C \cong \angle A$, $\overline{AB} \cong \overline{CB}$ by the converse of the isosceles triangle thm.

- Using Algebra
  - An isosceles triangle has a certain type of symmetry about a line through its vertex angle.
  - Theorem 22
    - If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.
    - If... $\overline{AC} \cong \overline{BC}$, $\angle ACD \cong \angle BCD$ Then... $\overline{CD} \perp \overline{AB}$, $\overline{AD} \cong \overline{BD}$



Name: ____________________

- Ex: What is the value of x?



$\angle A \cong \angle C$ (Thm 20)

$m\angle C = 54$

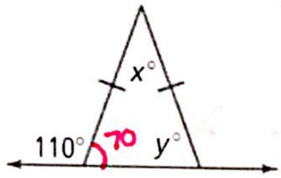
$\angle ADB \cong \angle CDB$ all right angles are $\cong$

$54 + 90 + x = 180$

$144 + x = 180$

$\boxed{x = 36}$

- Find the values of x and y.



$180 - 110 = 70$

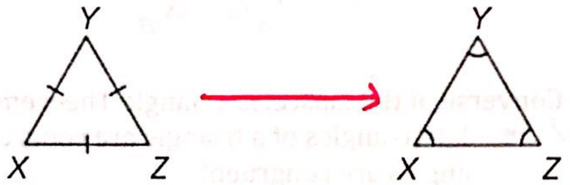
$\boxed{70 = y°}$ (base $\angle$s $\cong$)

$70 + 70 + x = 180$

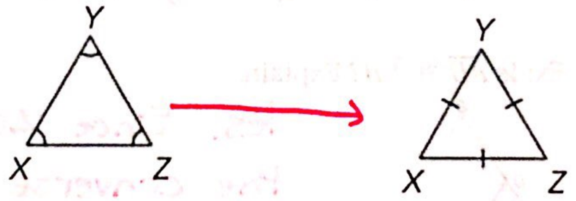
$140 + x = 180$

$\boxed{x = 40}$

- Finding Angle Measures
  - A corollary is a theorem that can be proved easily using another theorem.
  - Corollary to Theorem 20
    - If a triangle is equilateral, then the triangle is equiangular



  - Corollary to Theorem 21
    - If a triangle is equiangular, then the triangle is equilateral.



  - Ex: The equilateral triangle and the isosceles triangle share a common side. What is the measure of $\angle ABC$?

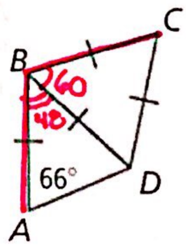
$\angle CBD = 60°$ (all $\angle$ in equilateral $\Delta \cong$)

$\angle ADB = 66°$ (Base $\angle$s $\cong$)

$2(66) + m\angle ABD = 180$

$132 + m\angle ABD = 180$

$m\angle ABD = 48$



$48 + 60 = m\angle ABC$

$\boxed{108 = m\angle ABC}$